Bosonic dark matter, misalignment mechanism, and cosmic Bose-Einstein condensation

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Dark Matter

- leading candidates:

1. WIMPs
2. Axions ~ $10^{-5}\,\mathrm{eV}$.
3. ALPs
4. Dark Photons

.....

To solve the strong CP problem, one introduces the $U(1)_{PQ}$ symmetry which is spontaneously broken

$$\begin{aligned} L &= -1/4g^2 Tr(G_{\mu\nu}G_{\mu\nu}) + \sum \bar{q}_(D_\mu \gamma_\mu + m_i)q_i \\ &+ \theta/32\pi^2 Tr G_{\mu\nu}\tilde{G}_{\mu\nu} + 1/2\partial_\mu a \partial^\mu a + a/(f_a 32\pi^2) Tr G_{\mu\nu}\tilde{G}_{\mu\nu}, \end{aligned}$$

$\theta + a/f_a \rightarrow 0$ relaxes to zero during QCD phase transition.

An example: the KSVZ axion

- one introduces an new complex scalar and a new heavy quark Q.

$$L_{Yu} = -fQ_L^\dagger \sigma Q_R - f^* Q_R^\dagger \sigma^* Q_R$$

$$V = -\mu_\sigma^2 \sigma^* \sigma + \lambda_\sigma (\sigma^* \sigma)^2$$

=> $\sigma = (v + \rho) exp(i\frac{a}{v}).$

$$\begin{aligned} U(1)_{PQ}: \quad a &\rightarrow a + f_a \alpha \\ \sigma &\rightarrow exp(iq\alpha)\sigma \\ Q_L &\rightarrow exp(iQ\alpha/2) Q_L \\ Q_R &\rightarrow exp(-iQ\alpha/2) Q_R \end{aligned}$$

Axion like particles

- alps arises due to compactification of the antisymmetric tensor fields

$$B = \frac{1}{2\pi} \sum b^i(x)\omega_i(y) + \dots ,$$

- the x are non-compact coordinate, y are compact coordinates.

Axion like particles

- the zero mode acquires a potential due to non-perturbative effects on the compactifying cycle.
- the effective Lagrangian in four dimension:

$$\mathcal{L} = \frac{f_{ALPs}^2}{2}(\partial a)^2 - \Lambda_{ALPs}^4 U(a)$$

ALPs

So if extra dimension is exist, alps seems inevitably.

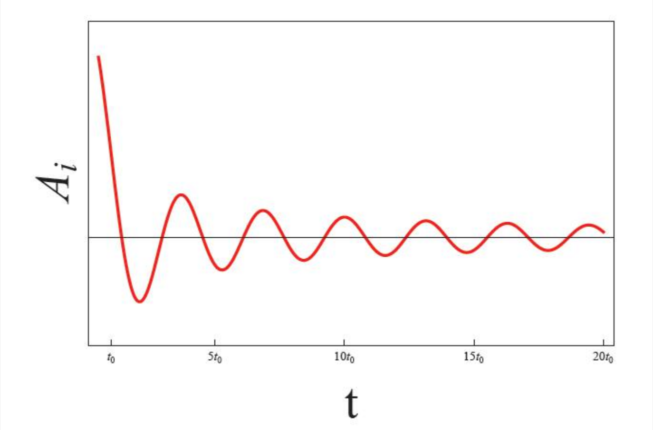
Question is: what is the mass?

Misalignment mechanism:

$$\partial_t^2 A + 3H\dot{A} + m_A^2 A = 0$$

during the radiation dominated era:

$$A'_i = C_1 J_{1/4}(M_{A'}t)/t^{1/4} + C_2 Y_{1/4}(M_{A'}t)/t^{1/4}$$



The cold axion (like) parties

- can be a major part of dark matter.

- have a very small velocity

$$V=[1/(t_1*m)]*[a(t_1)/a(t)]$$

$m<10^{(-6)}$eV. They are ultra light non-relativistic bosons.

Cosmic condensate

- cold alps phase space density is $\mathcal{N} \sim \frac{n_a}{k^3}$ >10^26, so they behave like condensate instead of thermal particles.

- the excited thermal states can only accommodate a given number of particles for a given temperature, the rest have to go to the ground state.

Axion/ALPs BEC

- the Lagrange in unperturbed flat FRW universe: $$\mathcal{L} = \frac{1}{2}\dot{a}^2 R^3 - \frac{1}{2}\partial_i^2 a R - \frac{1}{2} m^2 a^2 R^3 - \frac{\lambda}{4!} a^4 R^3$$

- we can derive the equation of motion

$$\partial_t^2 a - \frac{1}{R^2}\nabla^2 a + 3H\partial_t a + m^2 a + \frac{\lambda}{6} a^3 = 0$$

- PRL 103(2009) 111301 (QCD Axions)
- Int.J.Mod.Phys. A32, (2017) 1750051 (ALPs)
- PRD 95 (2017), 043541 (fuzzy dark matter) ...

Axion/ALPs BEC

alps are non-relativistic, so slow varying terms are interested. We factor out terms of order $e^{+/-imt}$, then we have a non-linear wave equation.

$$- i\dot{\psi} - \frac{1}{2mR^2}\nabla^2\psi - i\frac{3}{2}H\psi + \frac{\lambda}{8m^2}|\psi|^2\psi = 0 \ .$$

Axion/ALPs BEC

- if we consider gravitational in-homogeneity, then we have:

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - Gm^2\psi\int d^3x'\,\frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}$$

Perturbative regime

- from the wave function, we derive the generalized continuity equation:

$$\frac{\partial n_a}{\partial t} + \frac{1}{R}\frac{\partial (n_a v^i)}{\partial x^i} + 3Hn_a = 0$$

which is the same as the equation of point like CDM.

Perturbative regime

- the first order velocity equation is different:
$$\frac{\partial v^i}{\partial t} + Hv^i + \frac{\lambda}{8m^3}\partial_i n_a - \frac{1}{2m^2}\partial_i \frac{{\partial_i}^2\sqrt{n_a}}{\sqrt{n_a}} = 0$$

  comparing with point like CDM:
$$\frac{\partial v^i}{\partial t} + Hv^j = 0$$

we see two additional terms $-\frac{1}{2m^2}\partial_i \frac{{\partial_i}^2\sqrt{n_a}}{\sqrt{n_a}}$ $\frac{\lambda}{8m^3}\partial_i n_a$

which are due to quantum pressure and self interaction.

The jeans length

- combined all interactions (gravity, self-interactions), we find perturbation spectrum in k space:

$$\partial_t^2 \delta + 2H\partial_t \delta + \left(\frac{k^4}{4m^2R^4} - 4\pi G\rho - \frac{4\pi\sigma^{1/2}\rho}{m^3}\frac{k^2}{R^2}\right)\delta = 0 \ ,$$

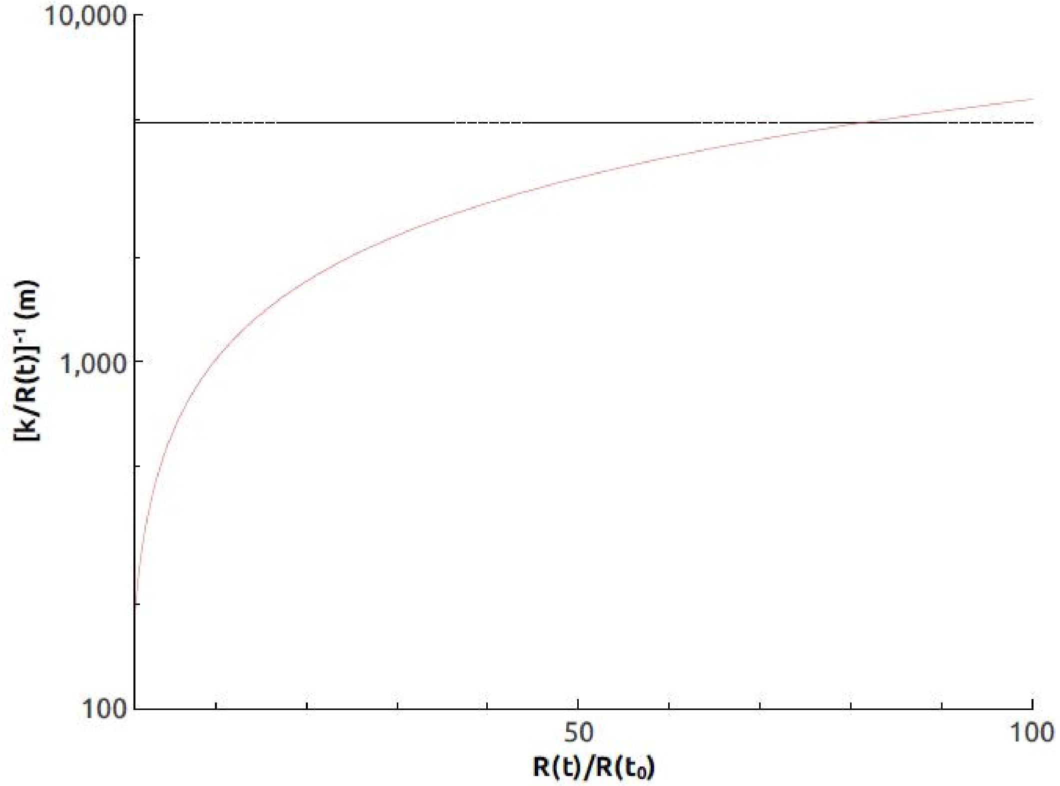


FIG. 2: The relative strength of the quantum pressure versus gravity and self-interaction versus gravity of QCD axions. The red line is the quantum pressure boundary above which gravity will dominate and the black dash line is the self-interaction boundary above which the gravity will dominate.

$$(k/R)^{-1} \sim (\sqrt{\sigma}/Gm^3)^{1/2}$$

$$(k/R)^{-1} \sim (16\pi m^2 G\rho)^{-1/4}$$

Int.J.Mod.Phys. A32, (2017) 1750051

None-linear evolution

- occupation number change

$$|\psi_k|^2 = -\frac{\lambda}{8m^2}\int \frac{d^3k'}{(2\pi)^3}\int \frac{d^3k''}{(2\pi)^3}\left[i\psi_{k'}\psi^*_{k''}\psi_{k+k''-k'}\psi^*_k + c.c\right]$$

- therefore the thermalization rate is:

$$\Gamma_k \sim \frac{8\pi G\, m^2\, n}{k^2}$$

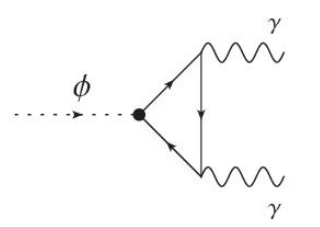
leading to a different cosmological structure formations.

Misalignment mechanism

- 1: Spin 0 axions, axion like particles
- 2: Spin 1 **Dark Photons**

ALPs

- decay channel



- decay rate:

$$\Gamma_\phi = \frac{\alpha^2}{4\pi^3\Lambda^2}M_\phi^3$$
$$= 6.53 * 10^{-10}\mathrm{s}^{-1}(\frac{M_\phi}{\mathrm{eV}})^3(\frac{\mathrm{GeV}}{\Lambda})^2 \quad .$$

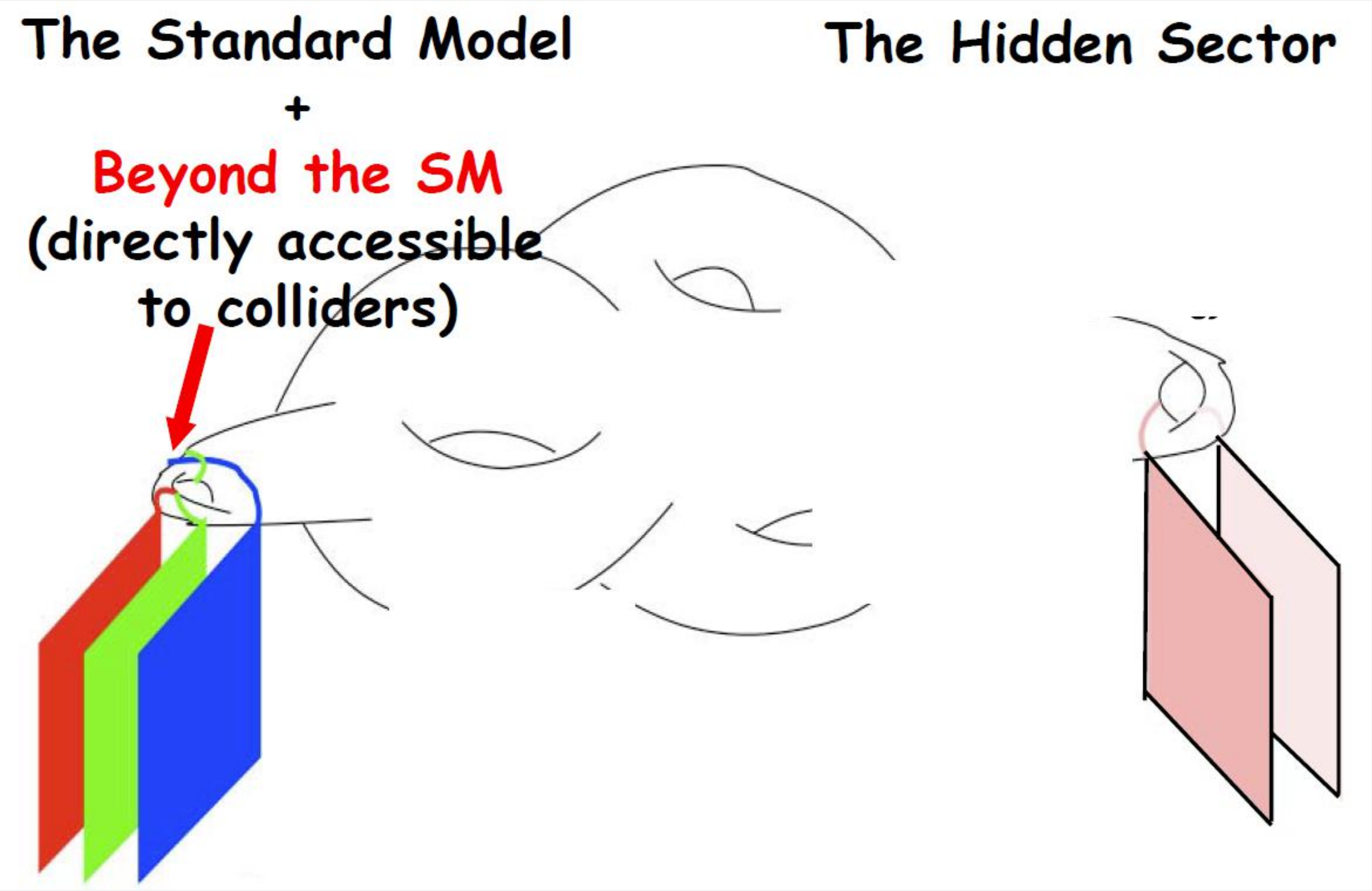
- QCD Axions: $M_\phi \sim m_{\pi^0} f_\pi/\Lambda \simeq 6.00 * 10^{-6}\mathrm{eV}(10^{12}\mathrm{GeV}/\Lambda)$

- Cosmological density ratio during the matter dominate era:

$$\frac{\rho_a}{\rho_c} \sim \frac{1}{2} M_\phi^2 \Lambda^2 \left(\frac{a(t_1)}{a(t)}\right)^3 / \rho_c \approx 3\pi G \sqrt{M}_\phi \Lambda^2 \sqrt{t}_e < 1,$$

Dark photons

hidden sector particles



Dark photons

$$\mathcal{L} = -\frac{1}{4}(F^{\mu\nu}F_{\mu\nu} + F'^{\mu\nu}F'_{\mu\nu} + 2\chi F'^{\mu\nu}F_{\mu\nu}) - \frac{M^2}{2}A'_\mu A'^\mu - e\bar{\psi}\gamma^\mu\psi A_\mu + \dots \quad ,$$

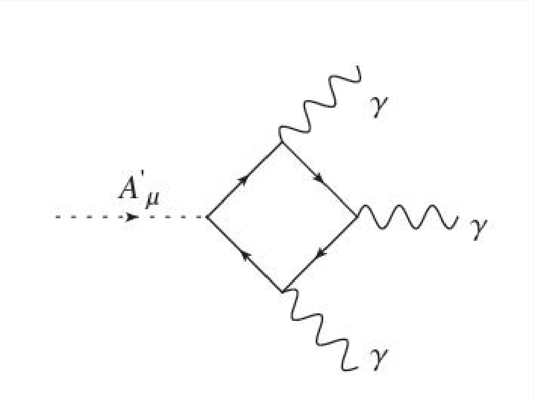
$$A_\mu \to A_\mu - \chi A'_\mu$$

$$\mathcal{L} = -\frac{1}{4}(F^{\mu\nu}F_{\mu\nu} + F'^{\mu\nu}F'_{\mu\nu}) - \frac{M^2}{2}A'_\mu A'^\mu - e\bar{\psi}\gamma^\mu\psi A_\mu - \chi e\bar{\psi}\gamma^\mu\psi A'_\mu + \dots \quad .$$

Dark photons

- Decay channel

$$\gamma' \to 3\gamma$$

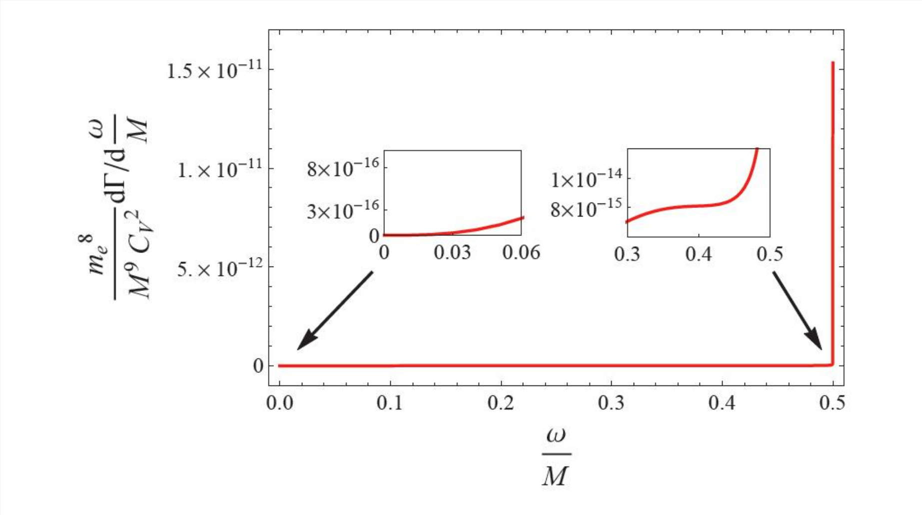


- Decay rate:

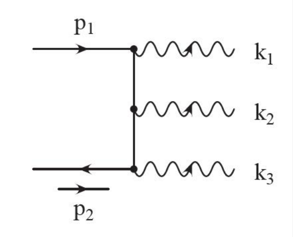
$$\Gamma_{A'} = 2.725 \times 10^{-15} \frac{C_V^2 M_{A'}^9}{m_e^8}$$
$$= 0.895 * 10^{-45} \text{seconds}^{-1} (\frac{M_{A'}}{\text{eV}})^9 C_V^2$$

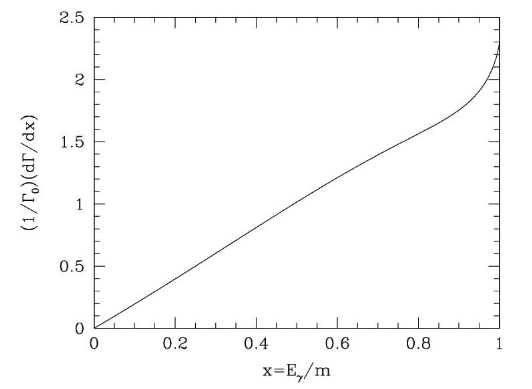
Dark Photons

- Spectrum:



- orthopositronium：





Dark matter halos

- up to 95% percent galaxy mass which is of order 10^(12) solar mass.

$$
\begin{aligned}
L_{A'} &= \frac{N_{A'} \times M}{\tau} = 0.95\frac{M_h}{\tau} \\
&= 151\frac{M_h}{M_\odot}(\frac{M}{\text{eV}})^9 C_V^2 \text{Watt} \\
&= 3.96 * 10^{-25}\frac{M_h}{M_\odot}(\frac{M}{\text{eV}})^9 C_V^2 L_\odot
\end{aligned}
$$

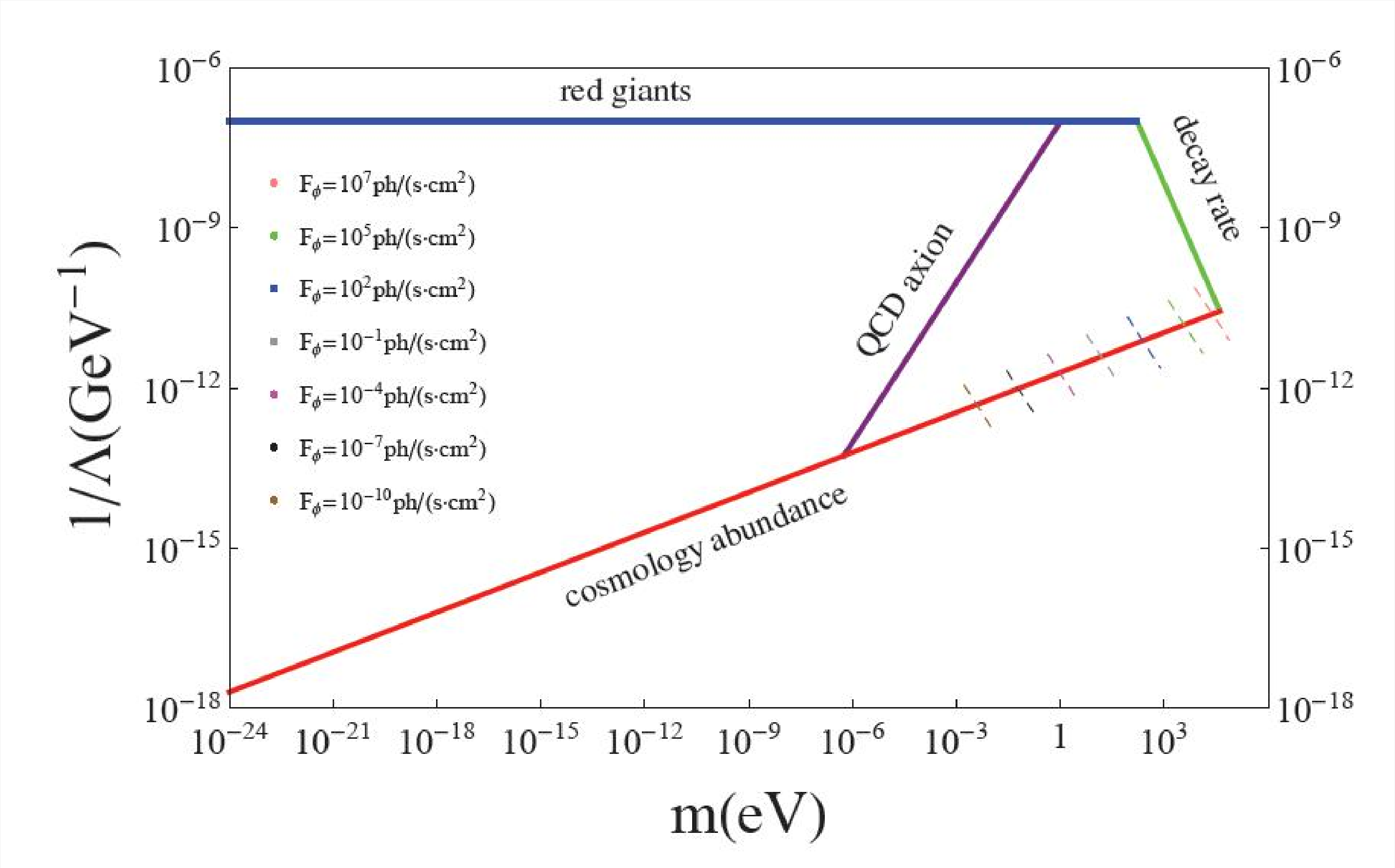
The distortion of spectrum

- Cosmology red-shift, Gravitational red-shift, Kinetic red-shift...

$$(1+\delta v/C)/(1-\delta v/C) \sim 2\delta v/C$$

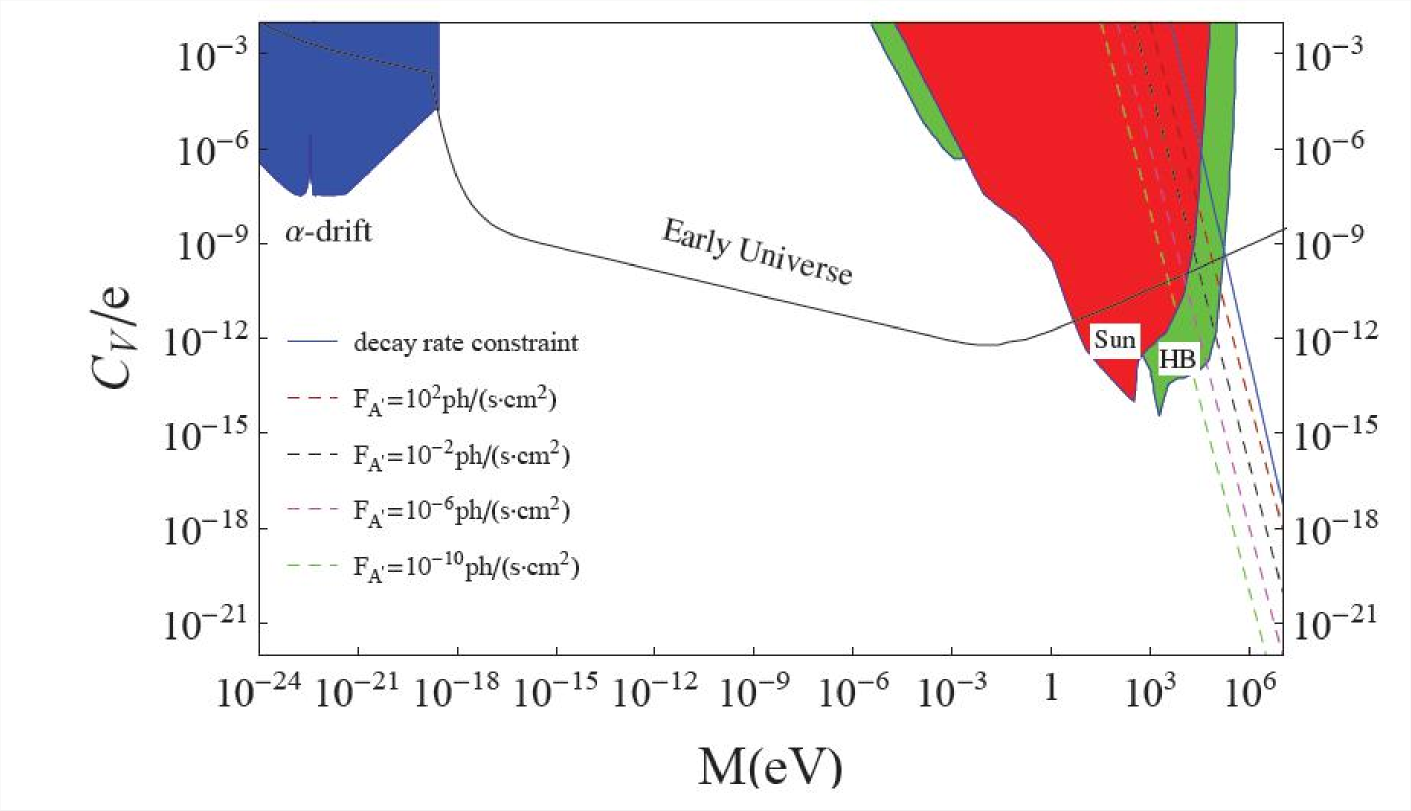
$$(1+z_k+z_c)^{-1}M/2.$$

The parameter space of ALPs



PRD 95 (2017), 075032.

The flux from the M31 received on the earth



PRD 95 (2017), 075032.

Dark photon detection

arXiv:1606.01492

Interactions in non-relativistic limit:

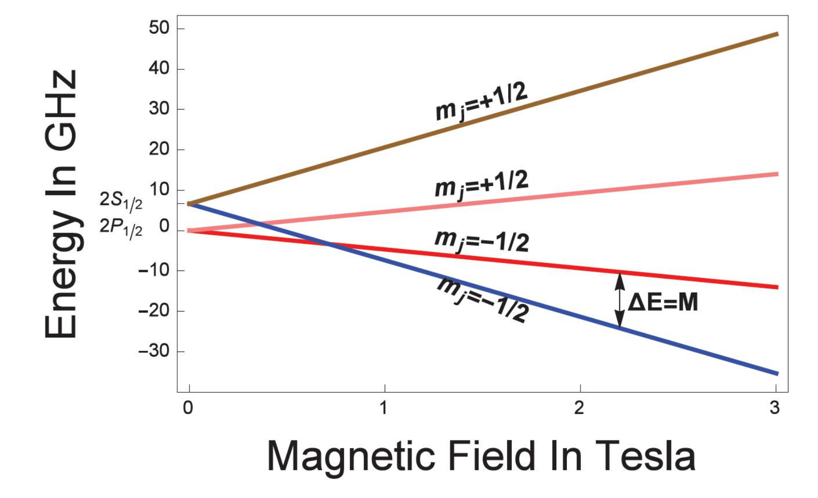
$$H = -\chi e(\vec{E}' \cdot \vec{x}) - [\chi e/(4M)]\vec{\sigma} \cdot \vec{B}' + ...$$

- in atomic scale，DP behaves like laser

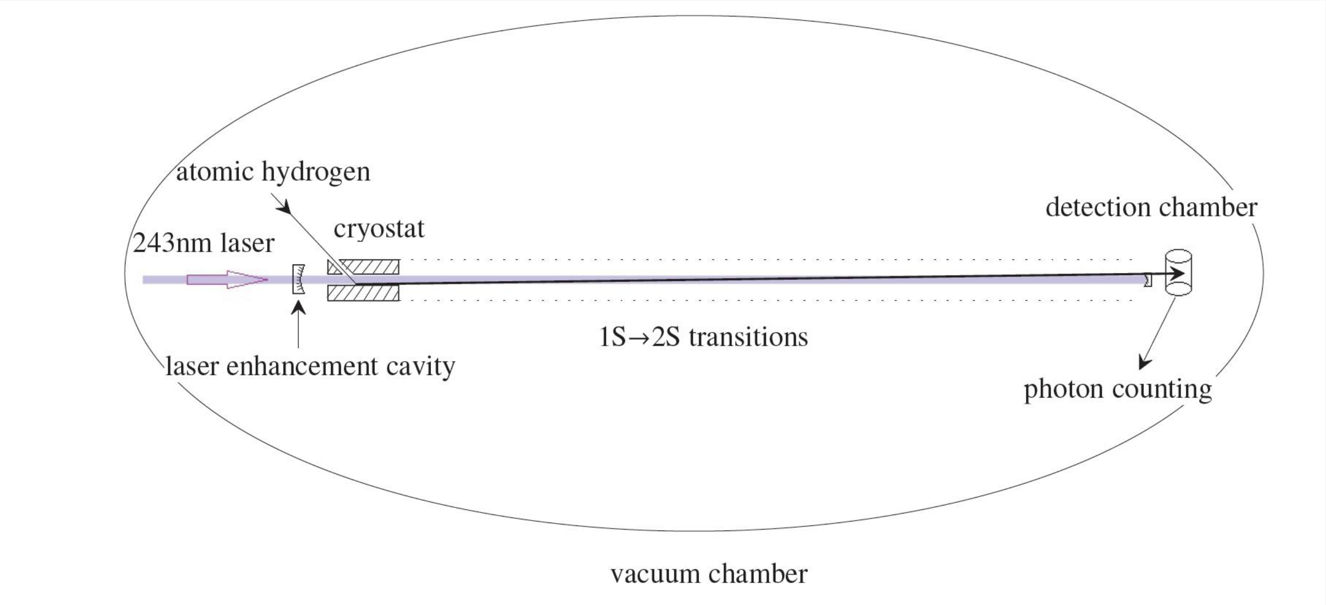
$$|\vec{E}| = \sqrt{2\rho_{cdm}} cos(Mt) \quad .$$

Dark photons

experiment detection:



Dark Photon Detection



$$RNt = \frac{4\pi}{3}\chi^2 e^2 I_{A'} a_0^2 N t = 1.93 * 10^8 \chi^2 N \frac{(t/\text{second})}{(M/\text{eV})}$$