Theories of QCD axion

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1. Symmetries

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- The U(1) problem and its solution by the theta term. [Weinberg (1975), 't Hooft (Phys. Report 1986)]

, $m_{\eta'} \le \sqrt{3}\, m_\pi$ near 1 GeV from contribution from $< G^a_{\mu\nu} \tilde{G}^{a\,\mu\nu} >$

The strong CP problem

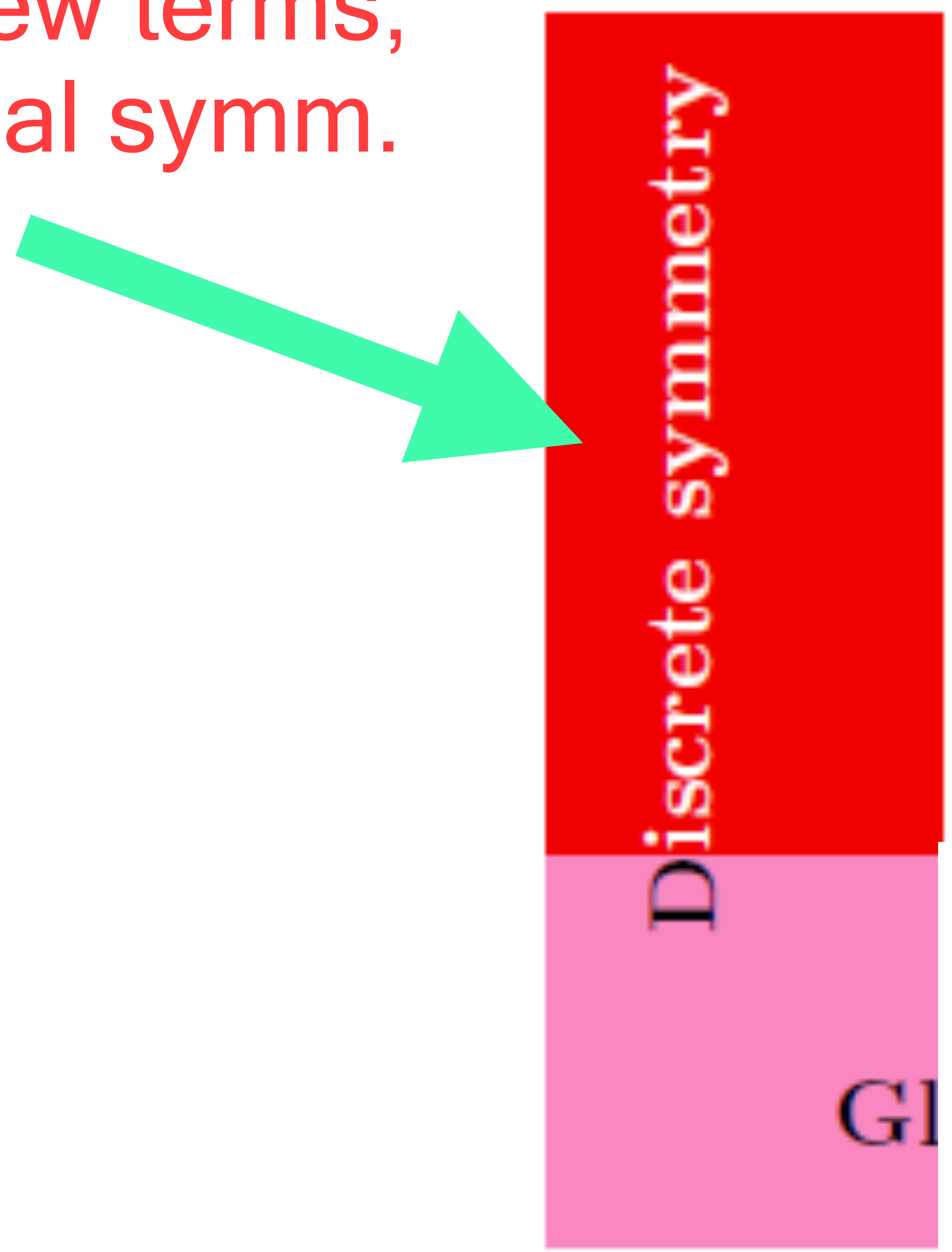
The theta term breaks CP symmetry and one expects that the NEDM is of order nucleon size times e. But, the upper bound is $O(10^{-26})$e cm, some 10 orders of magnitude away from anticipation. This is the strong CP problem. In the literature, three types were tried

(1) Massless up quark: up-quark is not massless.

(2) Calculable solutions: Nelson-Barr type.

(3) Axion solutions.

If we consider a few terms,
there can be global symm.



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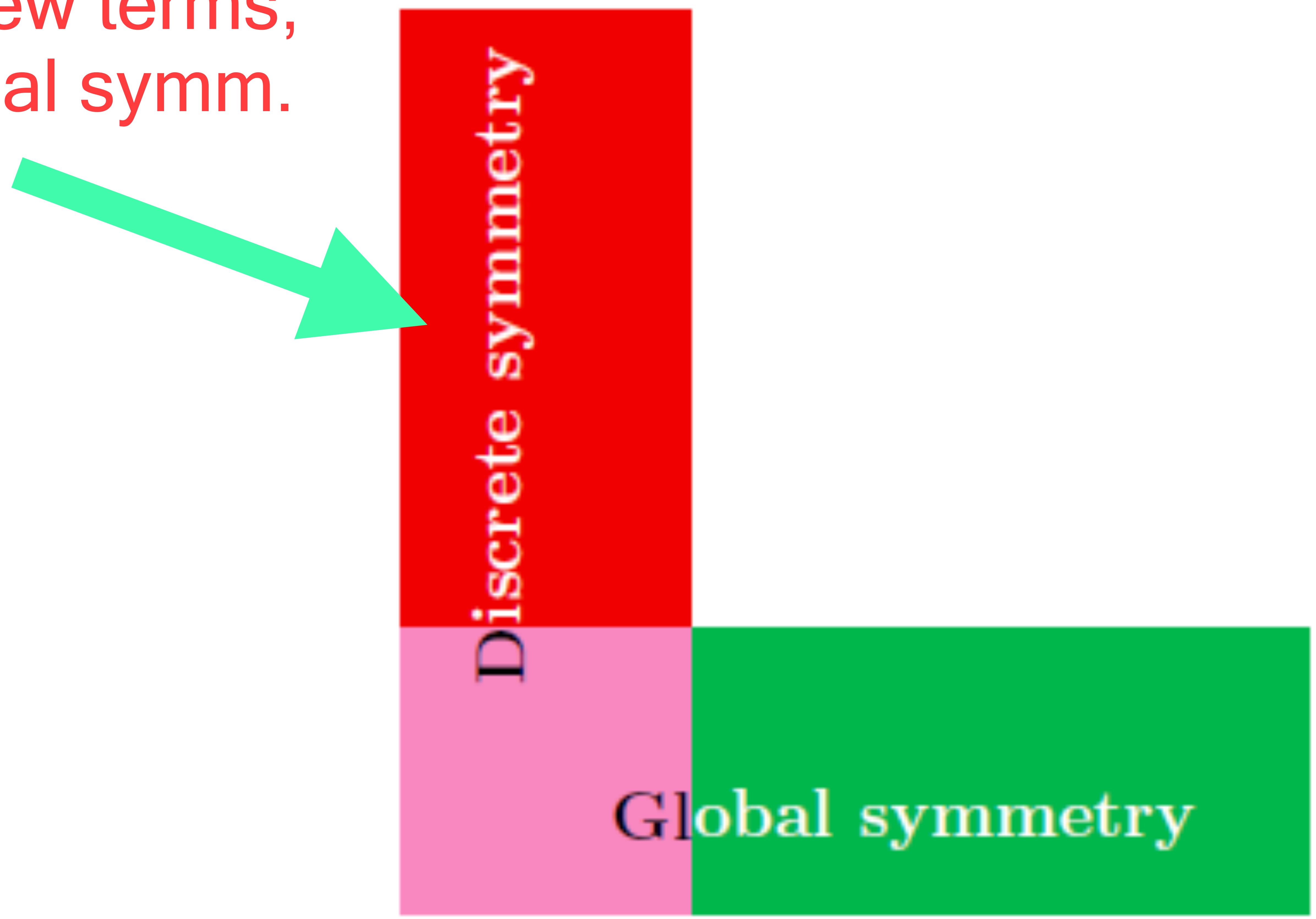

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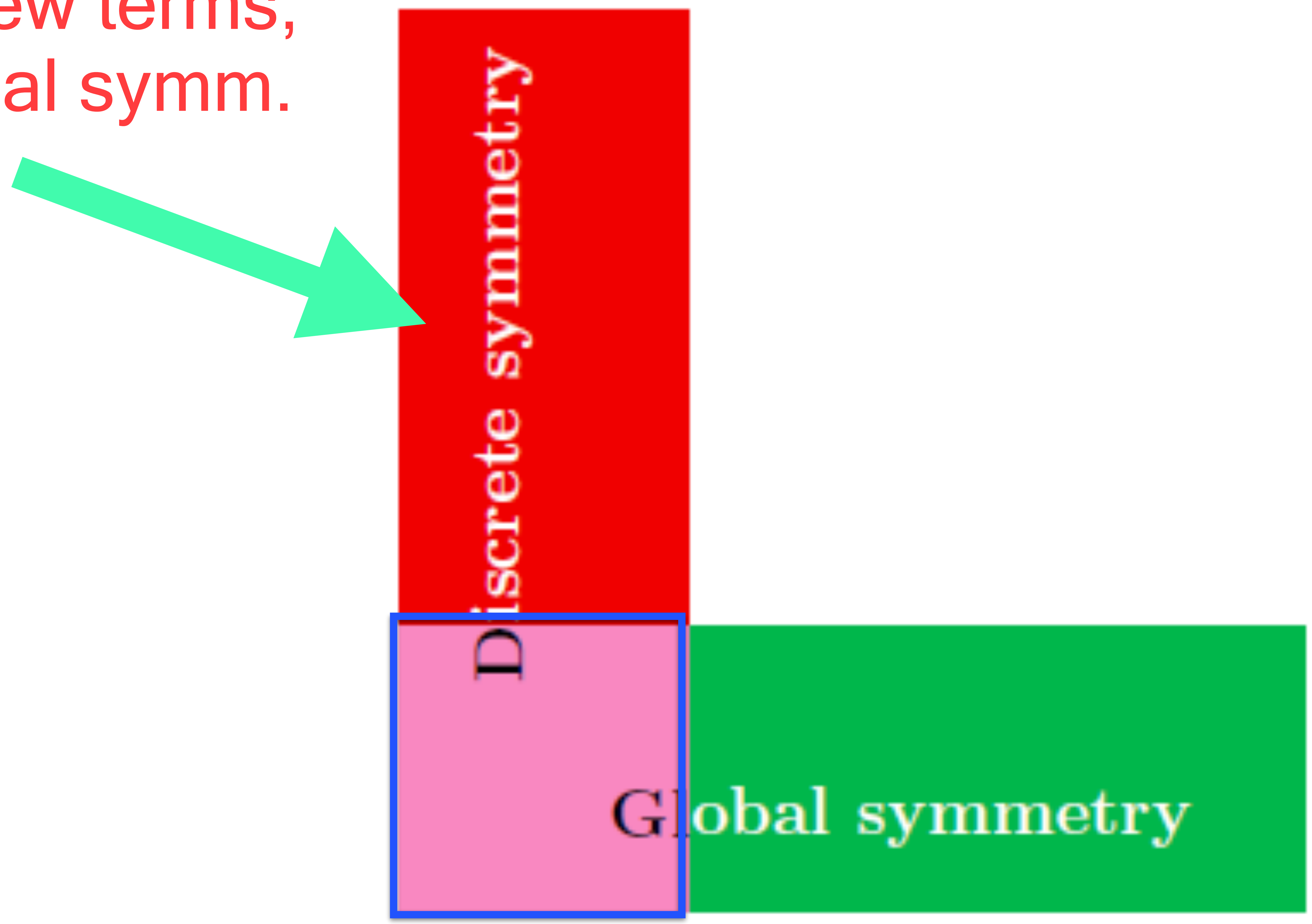

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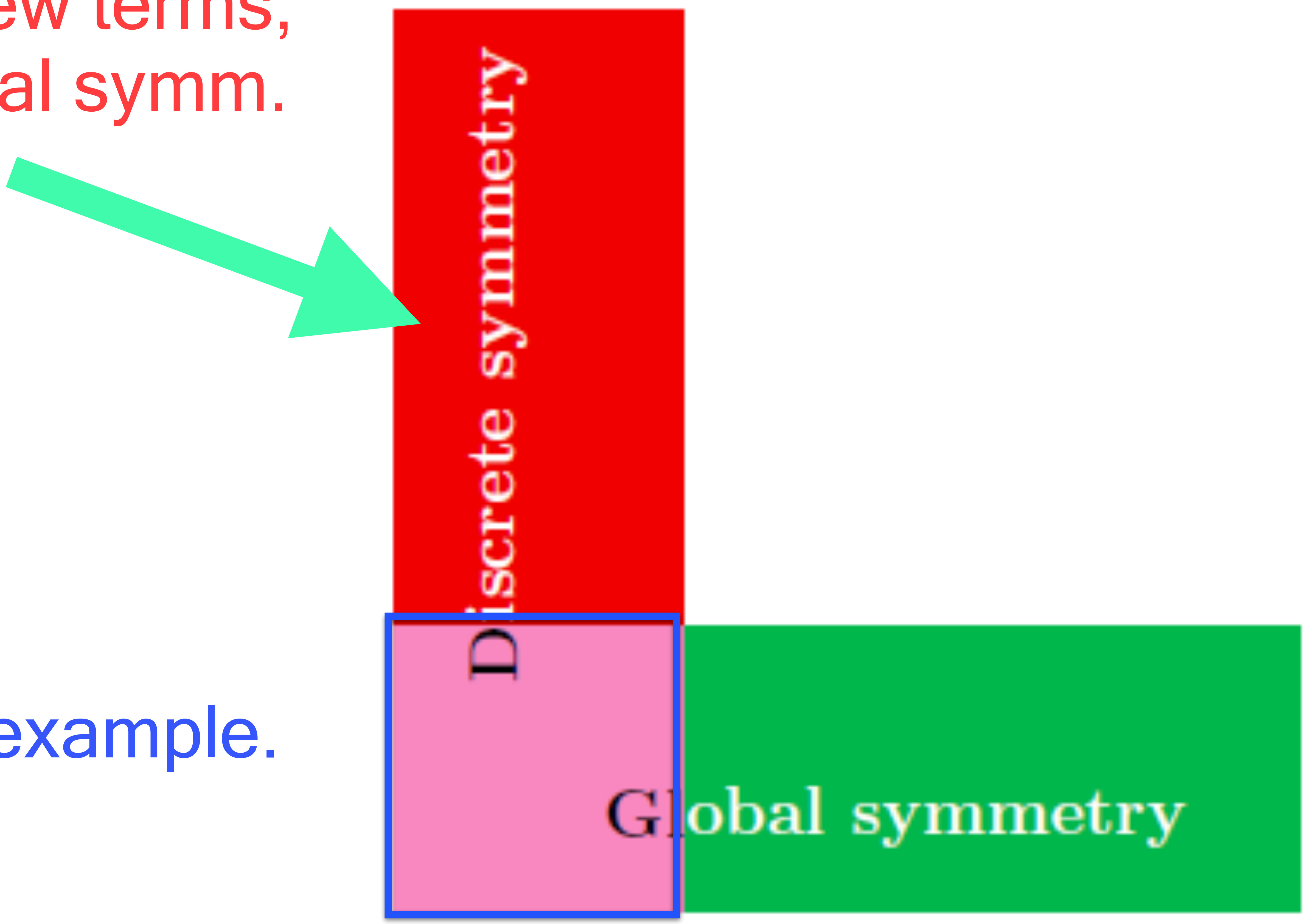

We start with an example.

$$V_{\rm W} = -\frac{1}{2}\sum_I m_I^2 \phi_I^\dagger \phi_I + \frac{1}{4}\sum_{IJ}\left[a_{IJ}\phi_I^\dagger\phi_I\phi_J^\dagger\phi_J + b_{IJ}\phi_I^\dagger\phi_J\phi_J^\dagger\phi_I\right.$$
$$\left. + c_{IJ}\phi_I^\dagger\phi_J\phi_I^\dagger\phi_J\right] + {\rm H.c.},$$

Not to have FCNC problem, one H doublet couples to u-type quarks and another H doublet couples to d-type quarks. With these Yukawa couplings, a general V_W with the reflection symmetry (ϕ_I to $-\phi_I$) breaks CP. But, if we keep all terms except the c_{IJ} terms, there appear a global symmetry: Peccei-Quinn symmetry.

But, with the quark global symmetry beyond the SM, in our case two, one must be a global symmetry. With this global symmetry of quark fields the phase appears in the theta term

$$\frac{\bar{\theta} - 2\alpha}{32\pi^2} \tilde{G}^a_{\mu\nu} G^{a\,\mu\nu}$$

Weinberg-Wilczek has shown that this phase field originally present in the Lagrangian develops a potential, and it is not an exact Goldstone boson but a pseudo-Goldstone boson. Phenomenologically, the PQWW axion is ruled out.

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$$\frac{\bar{\theta} - 2\alpha}{32\pi^2} \tilde{G}^a_{\mu\nu} G^{a\,\mu\nu} \qquad \bar{\theta} = \theta_{\rm QCD} + \theta_{\rm weak}$$

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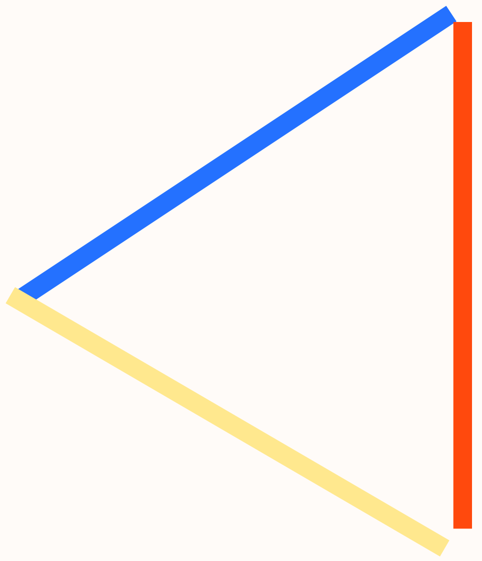

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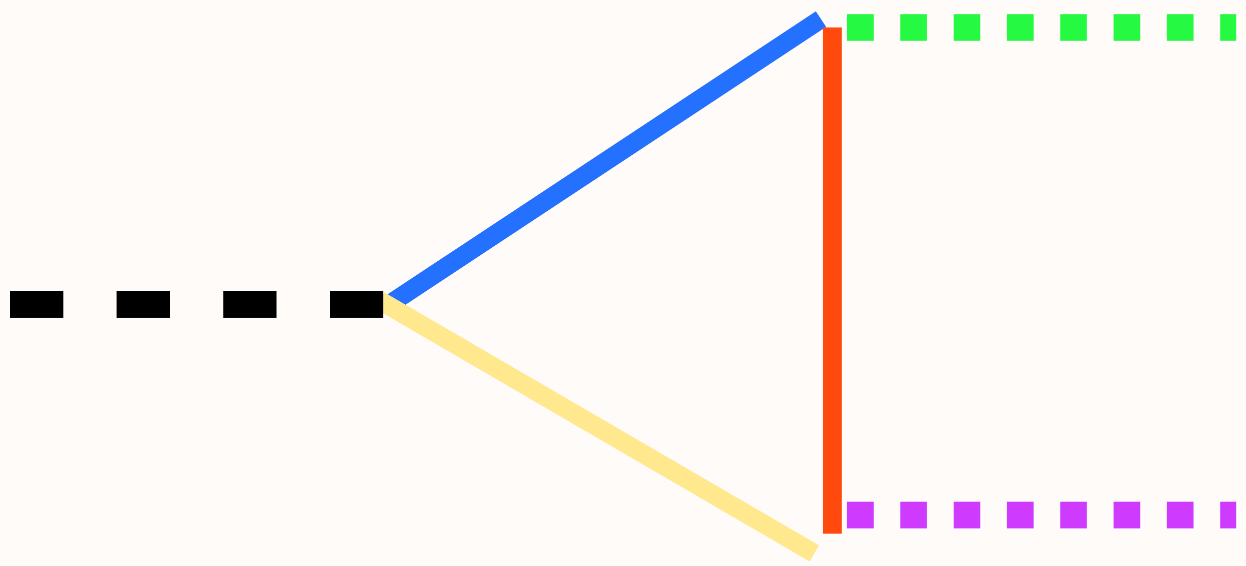

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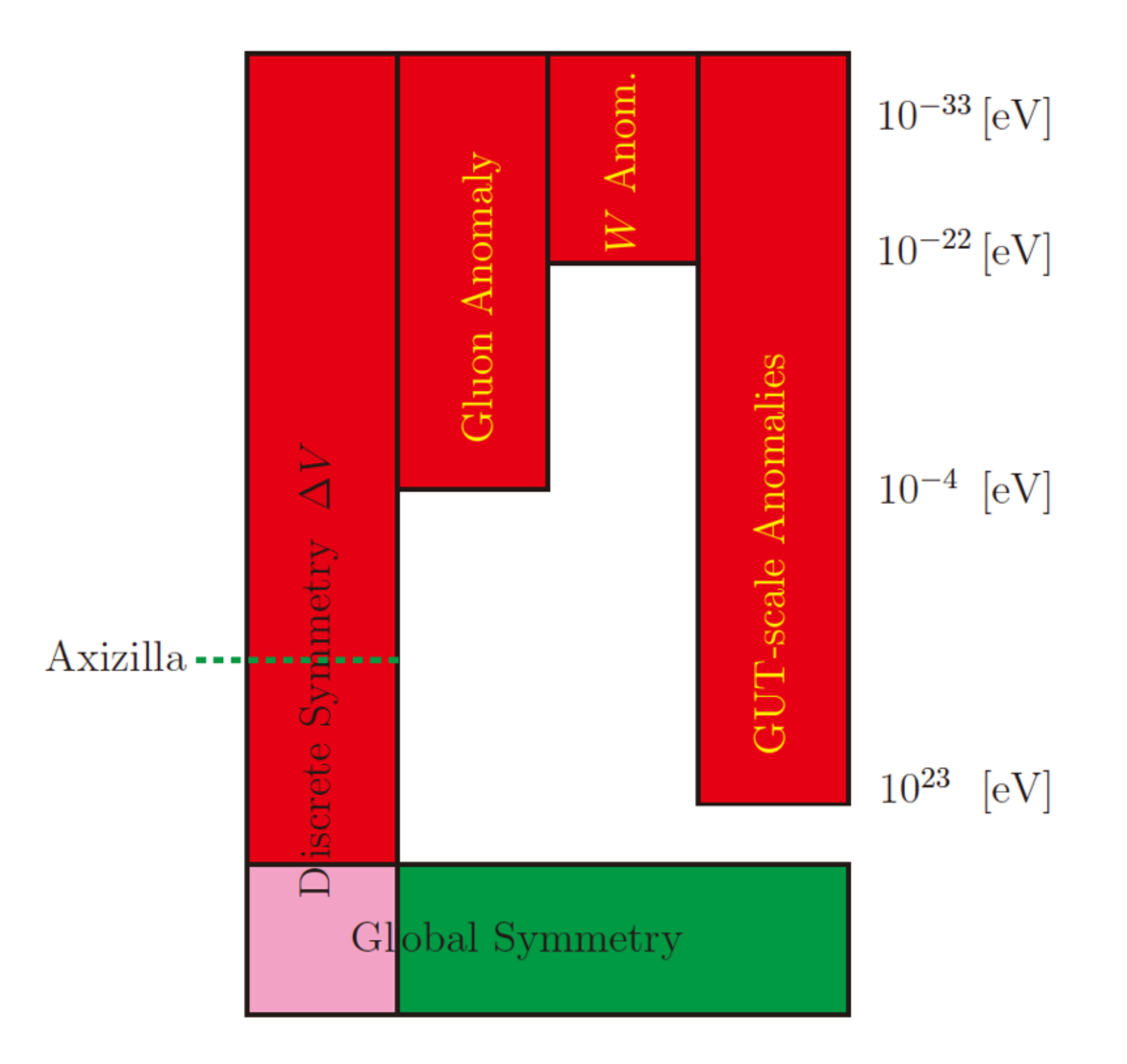
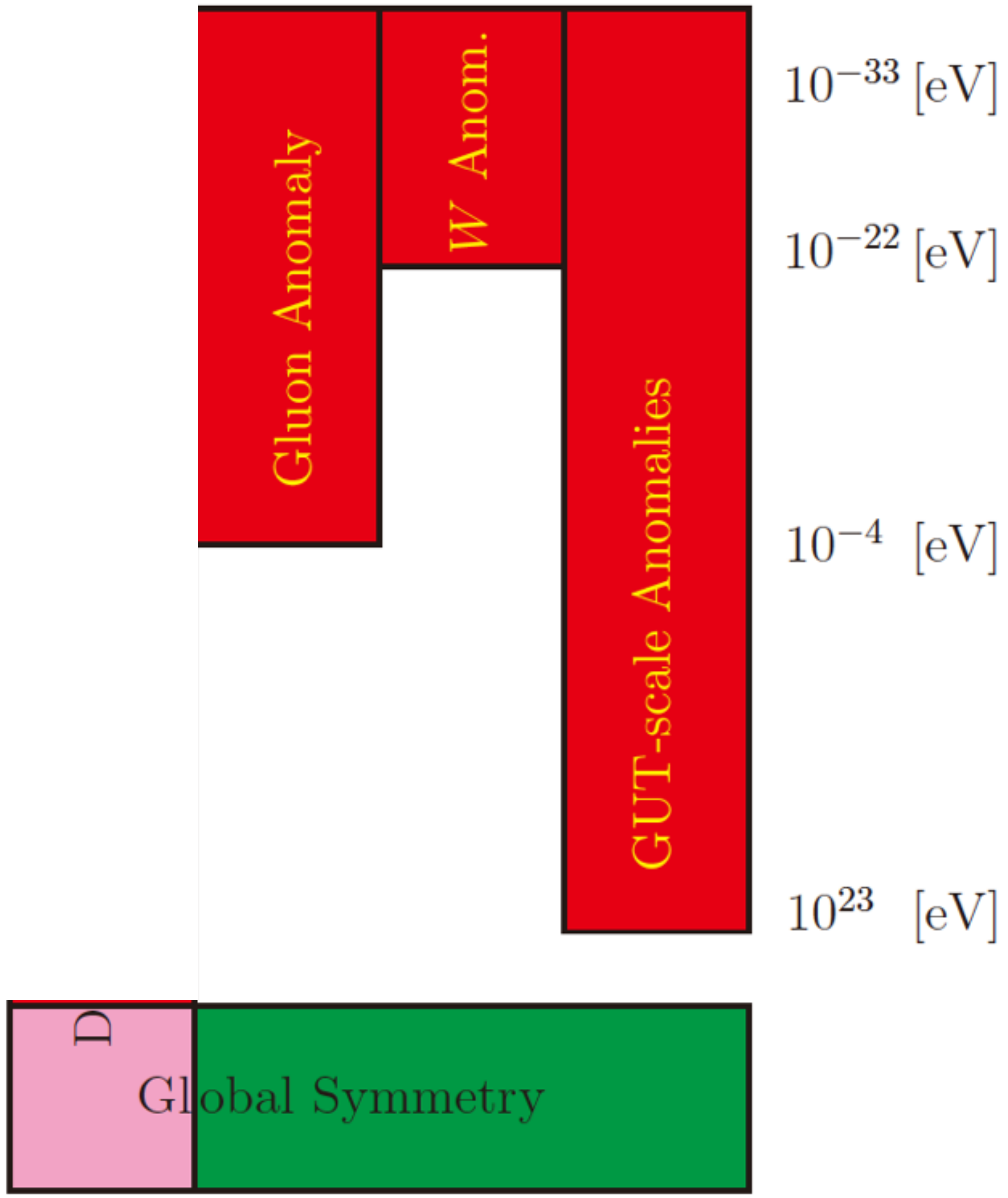
Axion solution has two important parameters

f_a =Intermediate scale, DW number.

2. “Invisible” axions



Gluon Anomaly
W Anom.
GUT-scale Anomalies
Discrete Symmetry ΔV
Axizilla
Global Symmetry
10^−33 [eV]
10^−22 [eV]
10^−4 [eV]
10^23 [eV]



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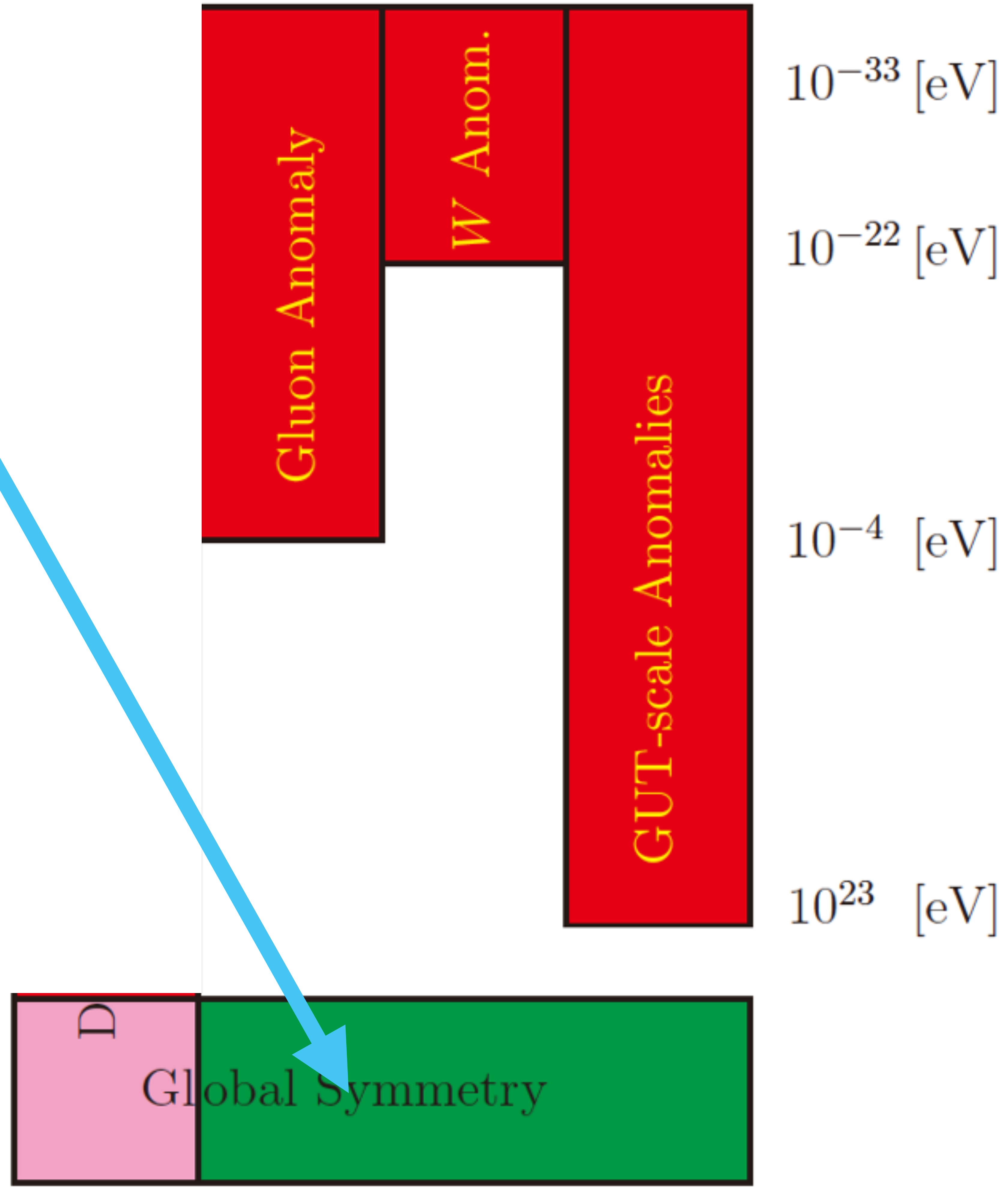
D
Global Symmetry

From the exact global symmetry.



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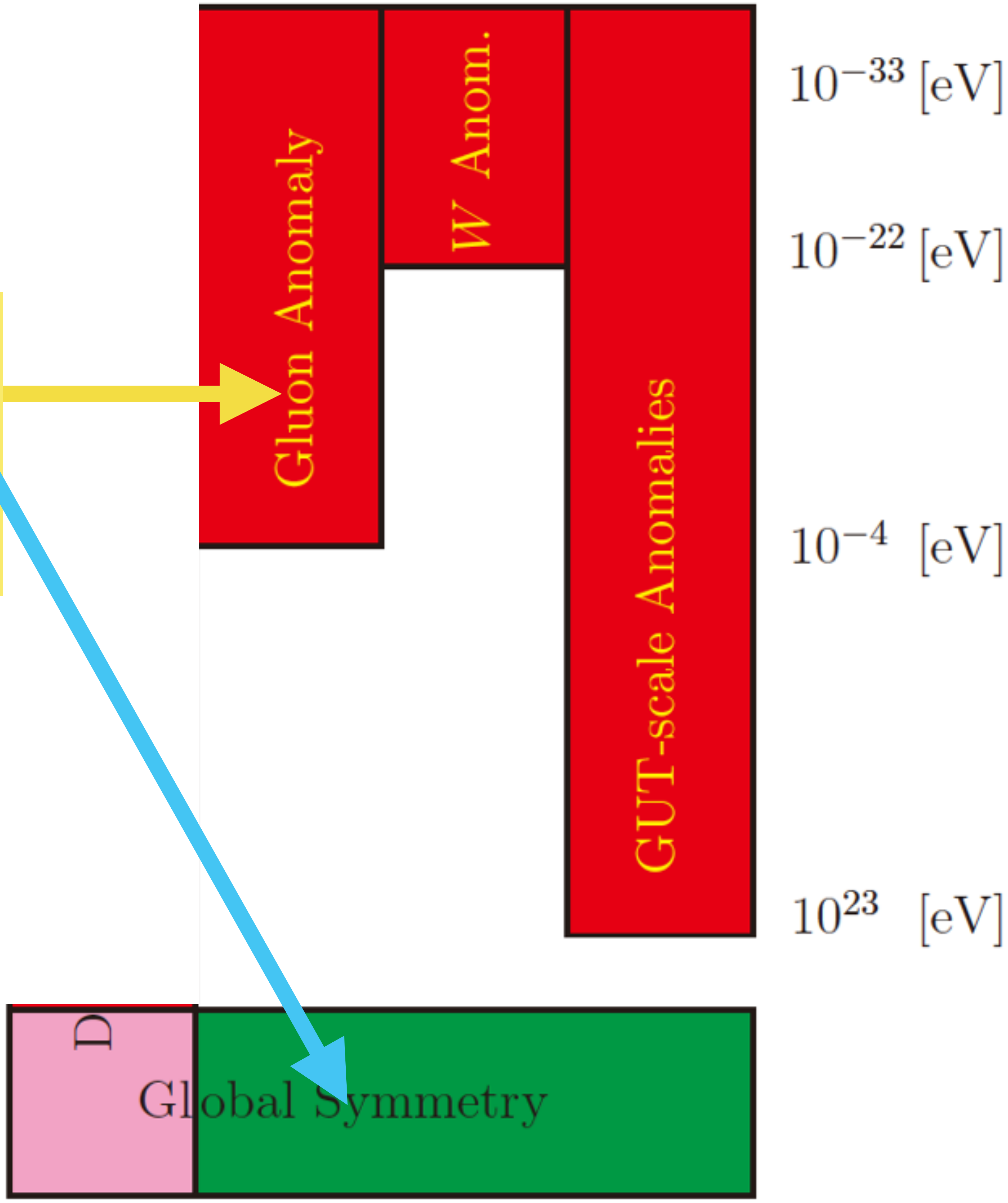
This anomaly breaks the PQ symmetry.



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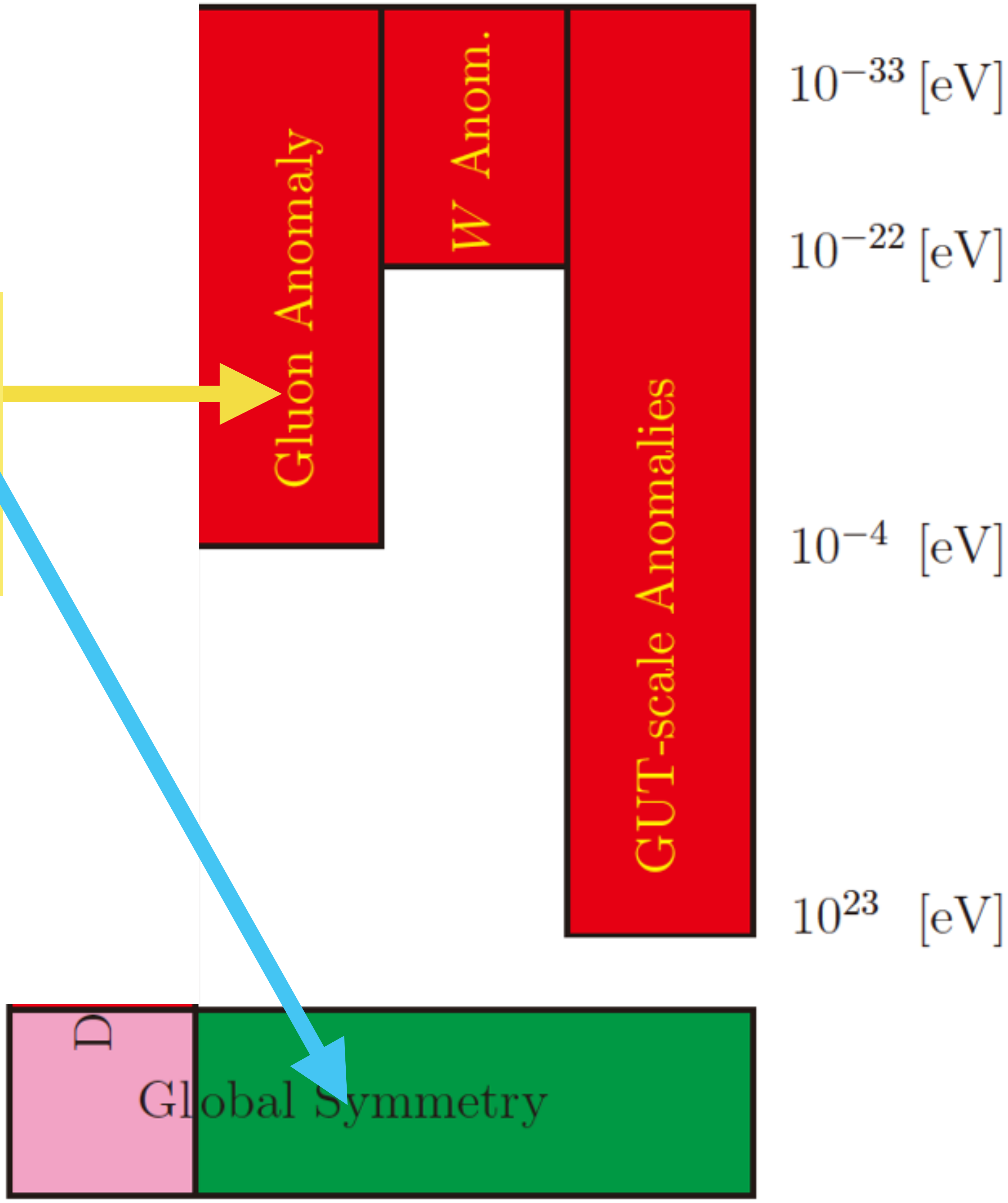
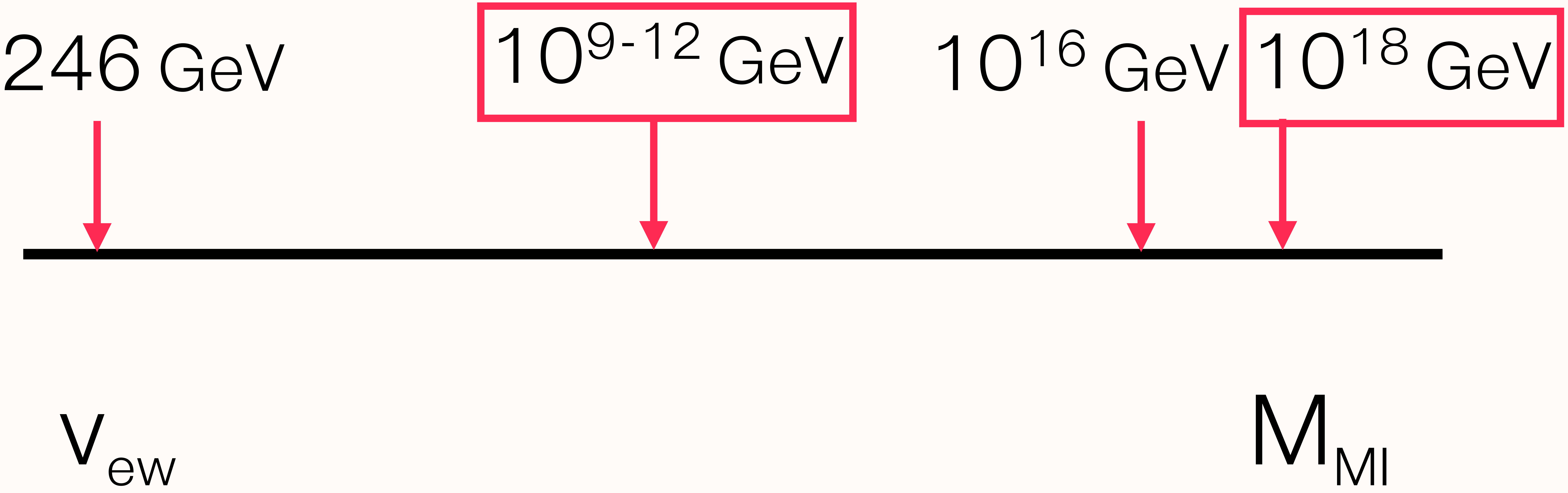
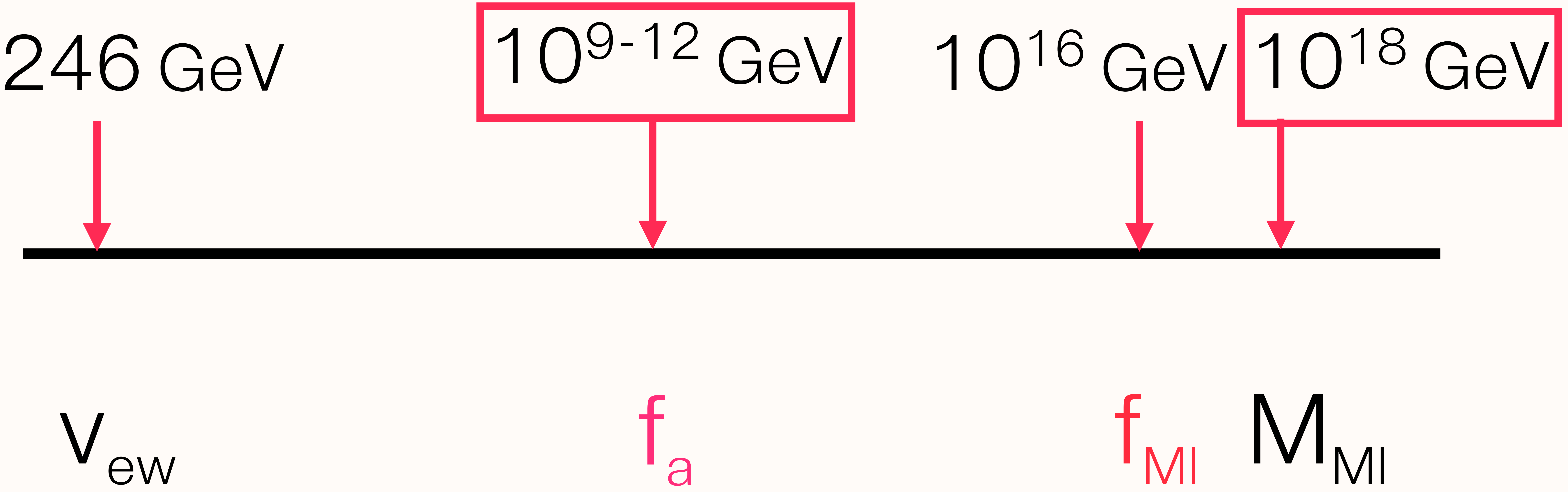


VEV of scalar phi gives the f_a scale.


246 GeV
10^{9-12} GeV
10^{16} GeV
10^{18} GeV
V_{ew}
M_{Ml}



246 GeV
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v_{ew}
f_a
f_{MI}
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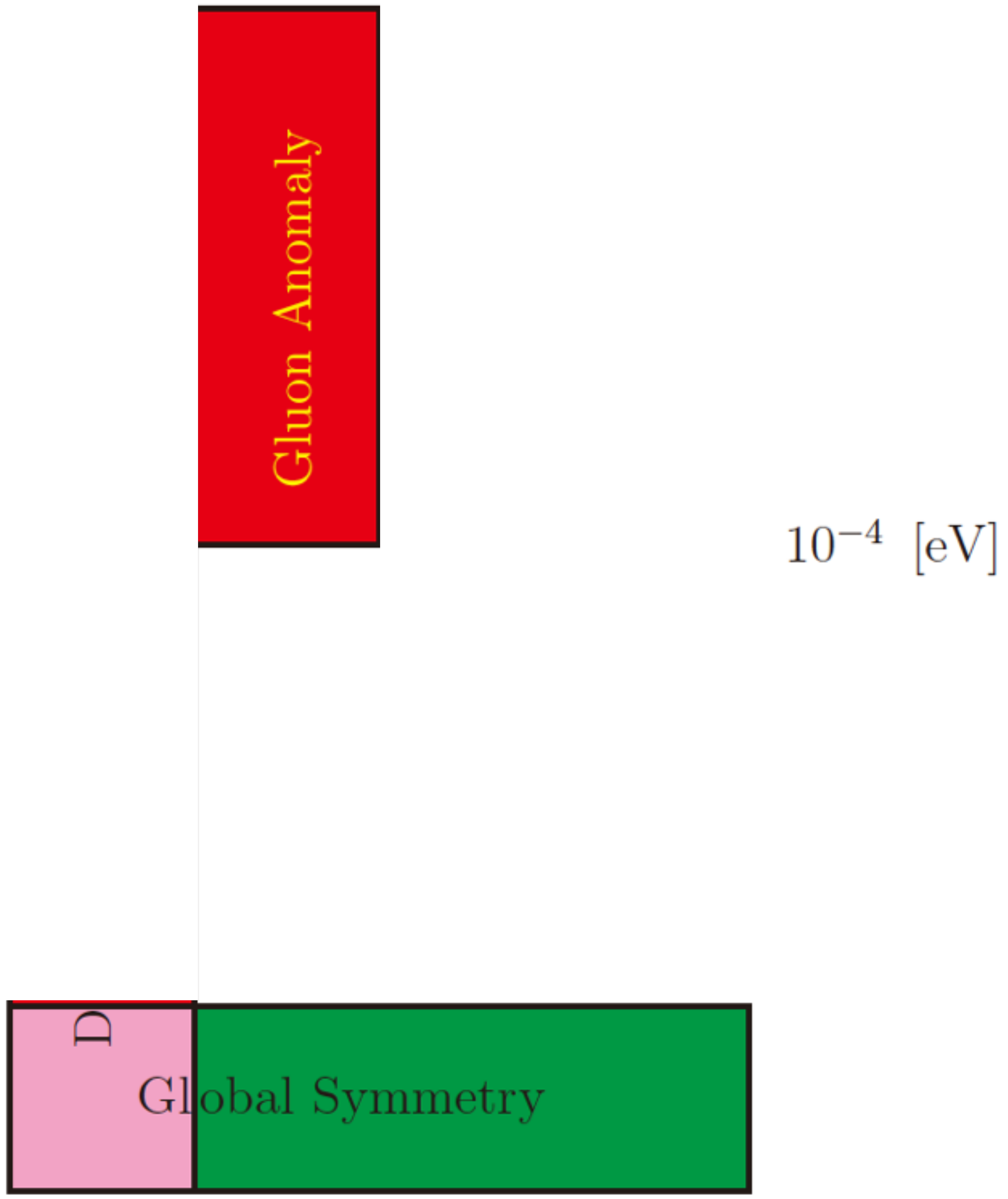
This is possible only after having a spontaneously broken global symmetry far below the Planck mass scale. We will come back to this point later.

Scale of f_a : KSVZ axion



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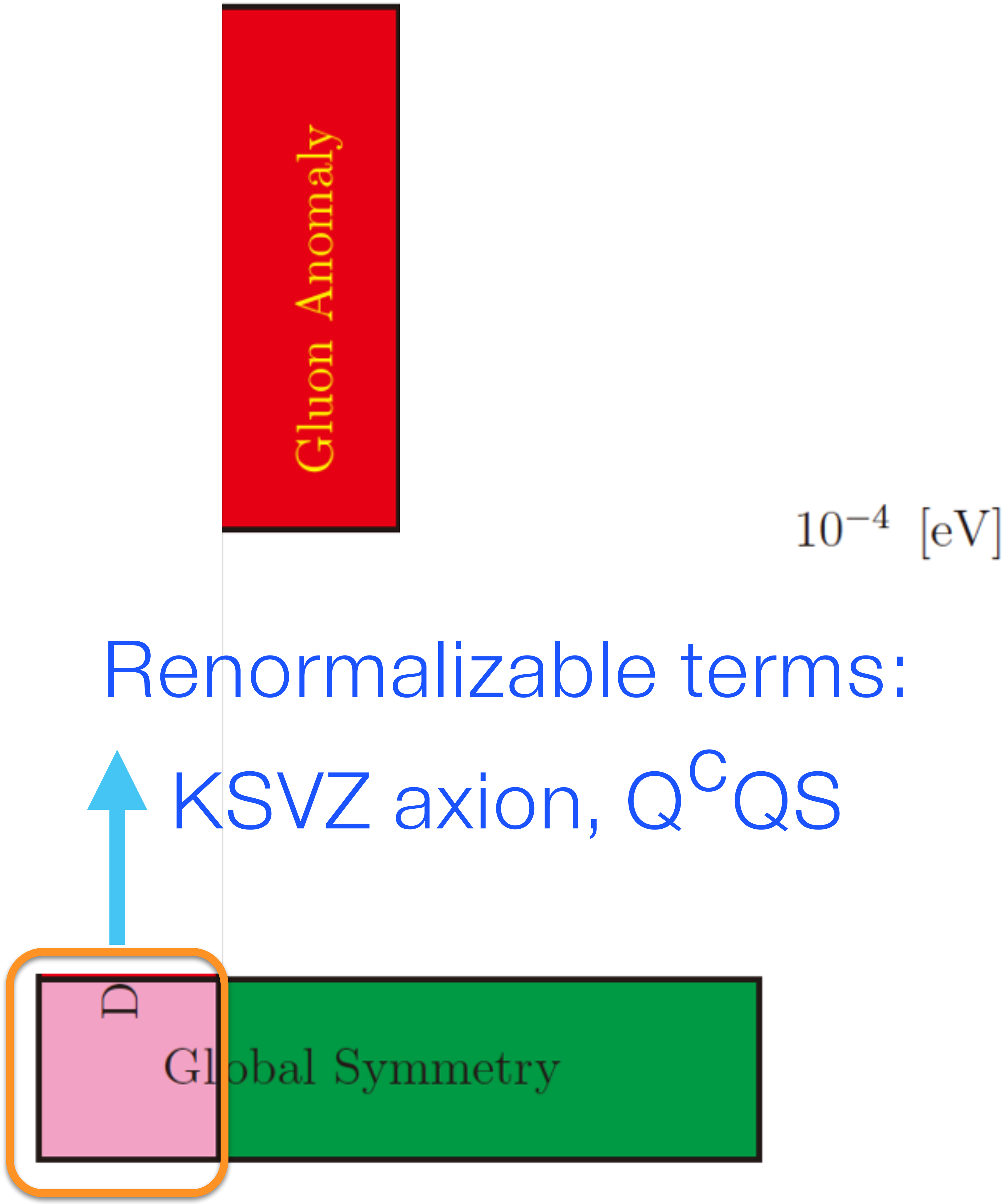

Renormalizable terms:

KSVZ axion, Q^cQS

Scale of f_a :

DFSZ axion

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DFSZ axion

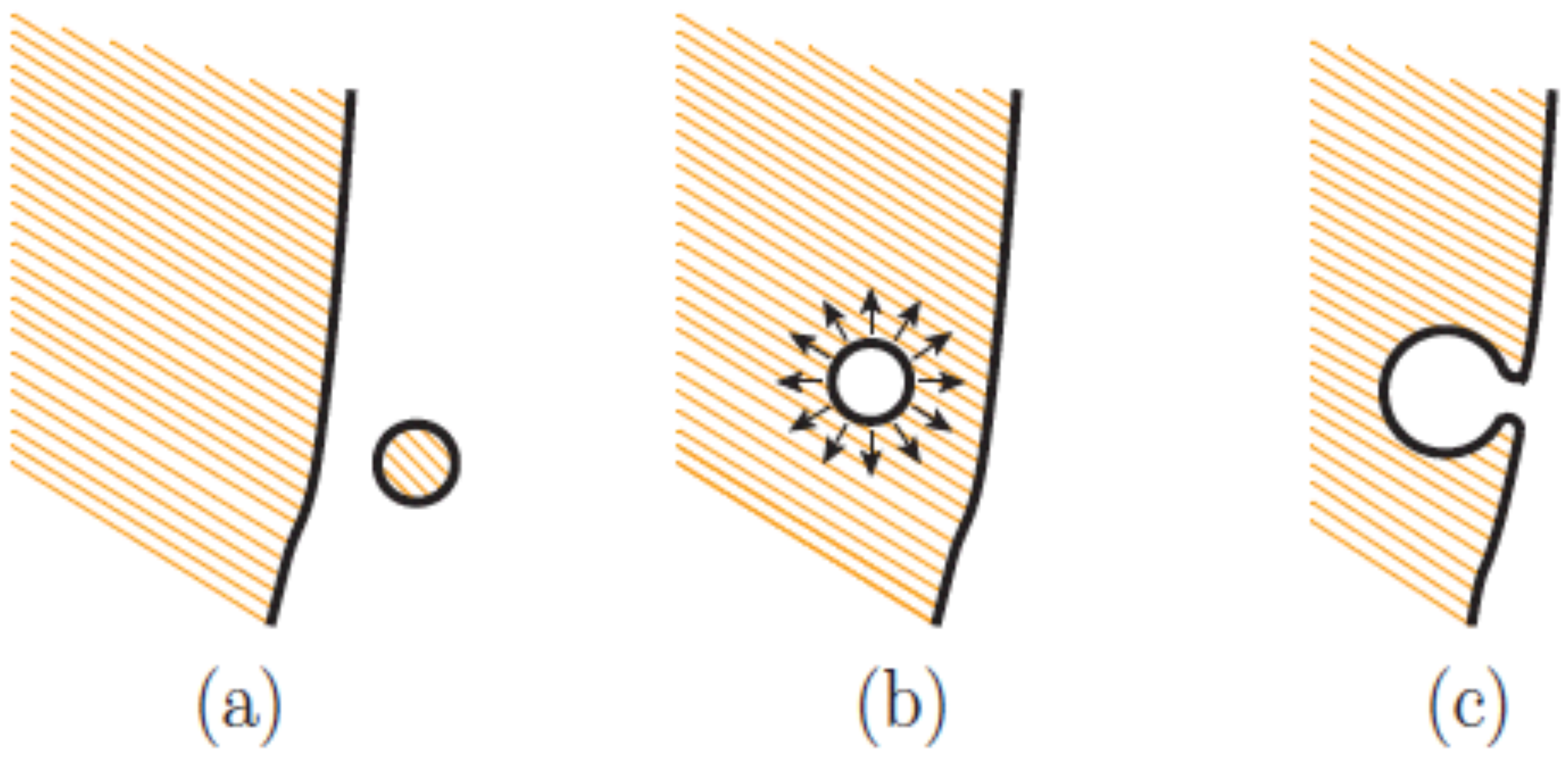
Renormalizable terms with fine-tuning:

V for DFSZ axion, $H_1 H_2 S^2$, $|S|^4$

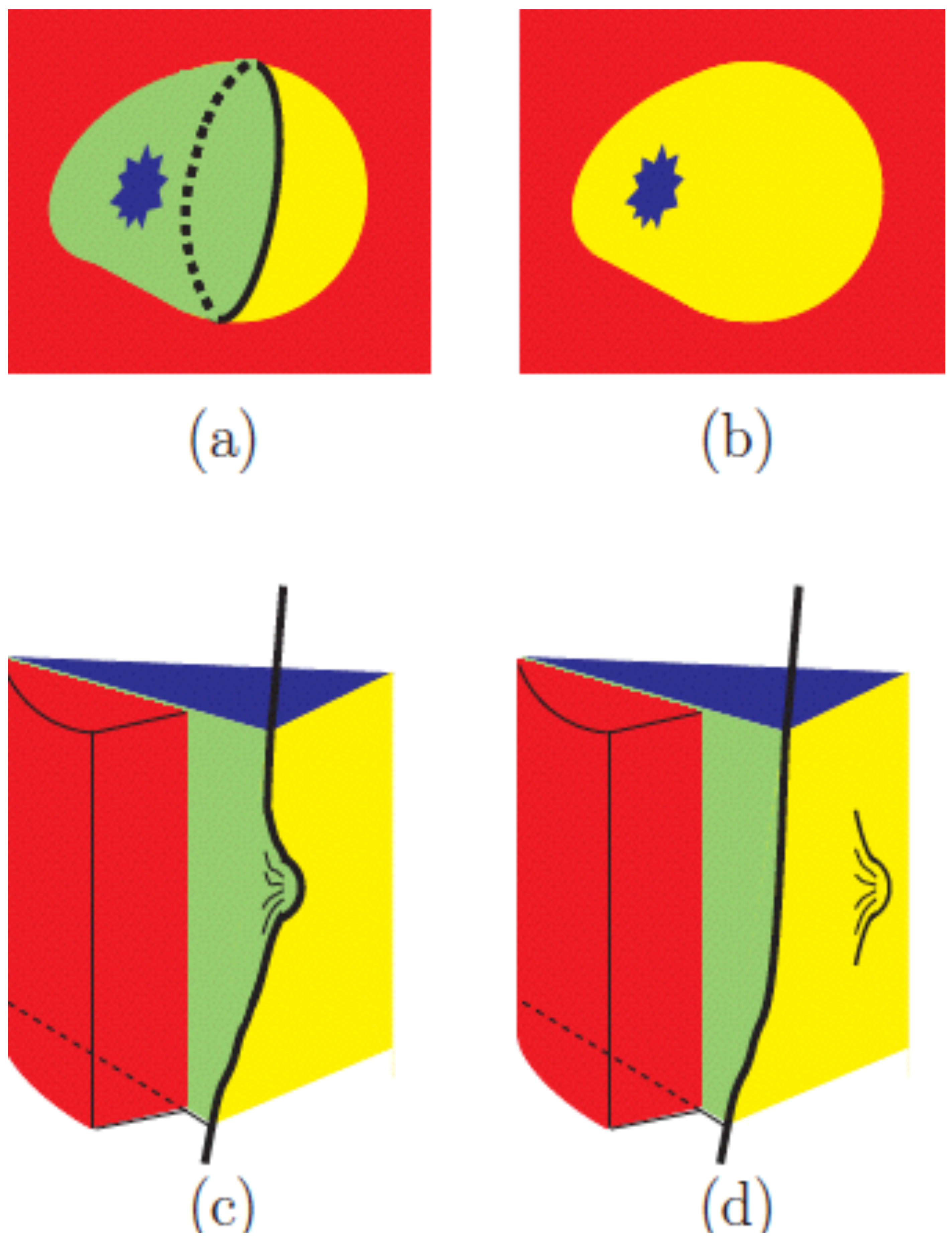
Non-renormalizable terms with SUSY: no fine-tuning

W for DFSZ axion, $H_1 H_2 S^2$

Domain-wall problem



Vilenkin-Everett (1982);
Barr-Choi-Kim (1987)



Sikivie (1982)

N_{DW}=1 needed

Top-down approach, using string compactification

1. The global U(1) is broken at the axion window.
2. DW number given here.
3. By giving a VEV to Q_{PQ}=1 field, we obtain N_{DW}=1.
4. Example: 1 heavy quark model. But, effectively, all PQ charged quarks should add up their contributions to makeN_{DW}=1.
5. Anomalous U(1) gauge symmetry.
6. Choi-Kim mechanism: with hidden sector force. Anomalous U(1) becomes global U(1) below the GUT scale.

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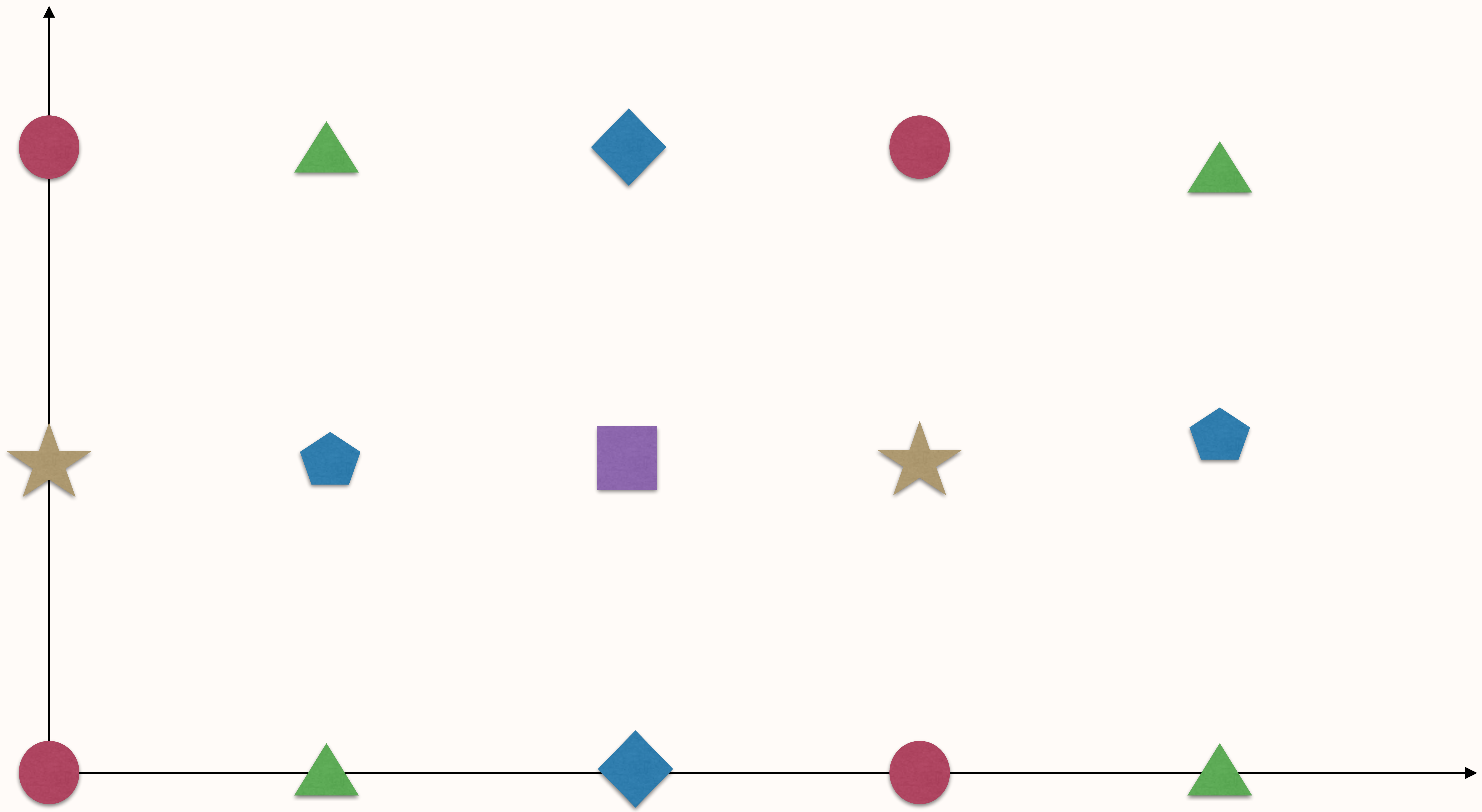
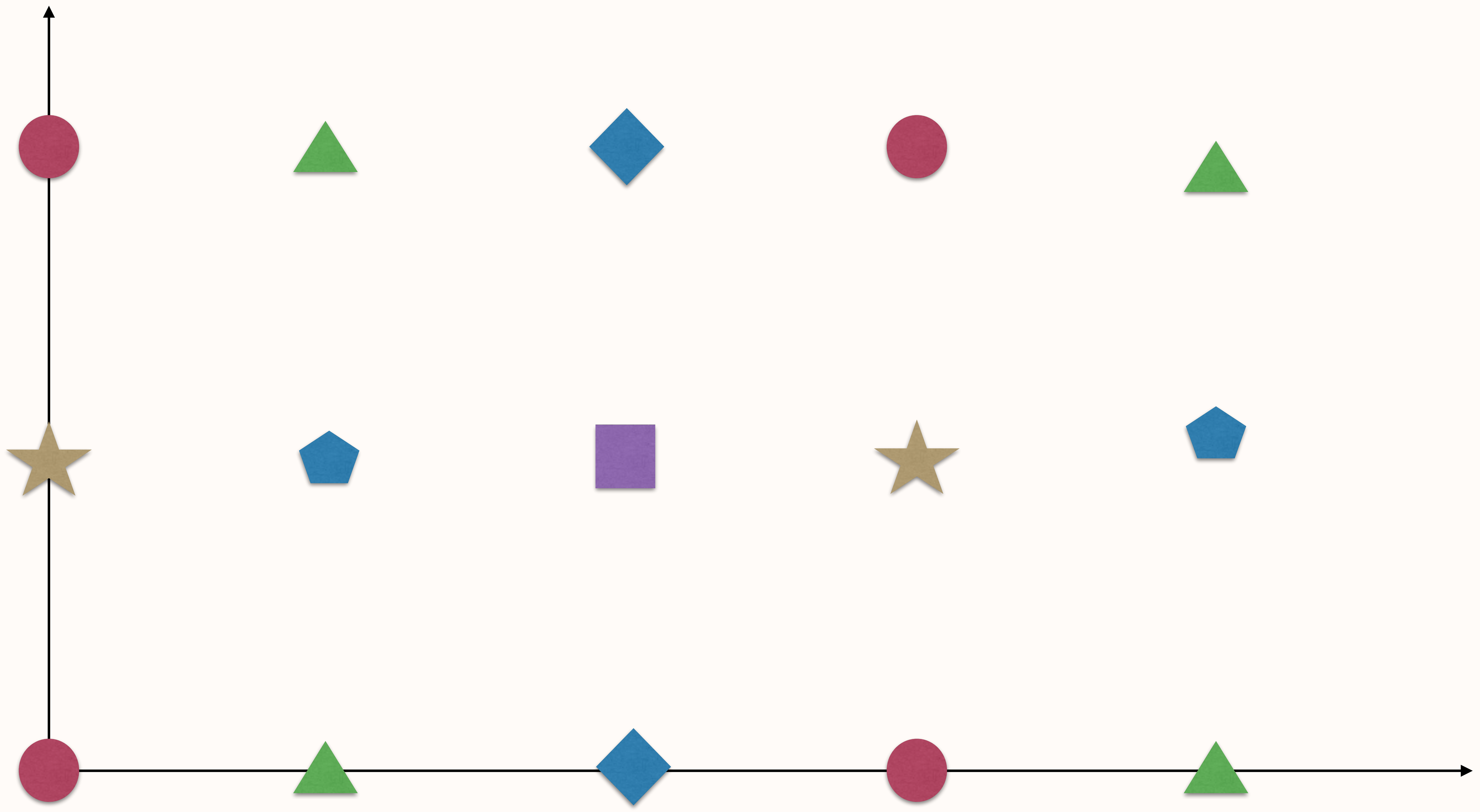
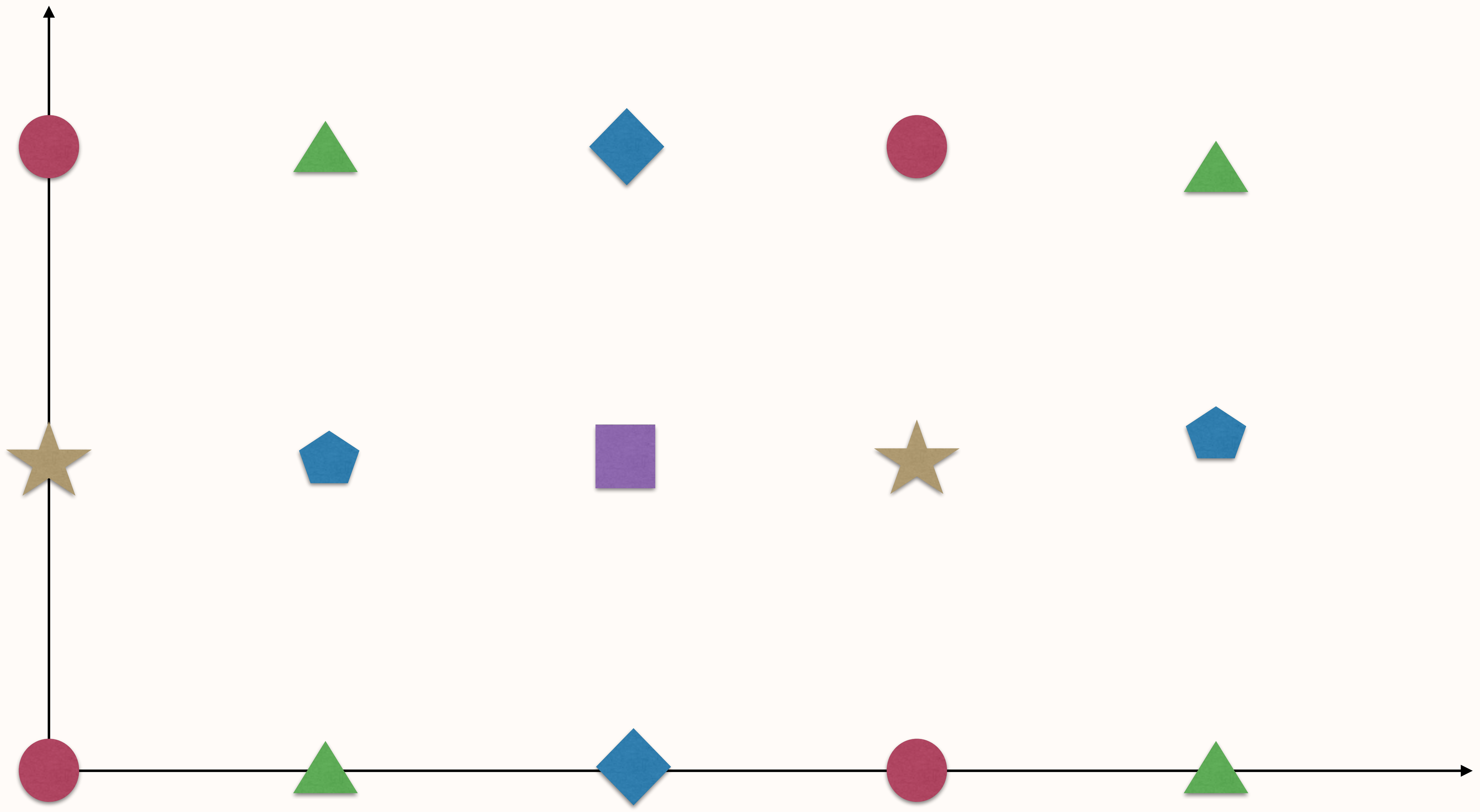
Better way than Lazarides-Shafi: we need a Goldstone boson direction



For the center of GUT group, Lazarides-Shafi (1982).
But, the following ideas are more widely applicable.



Choi-Kim, PRL55 (1985) 2637 with two confining forces



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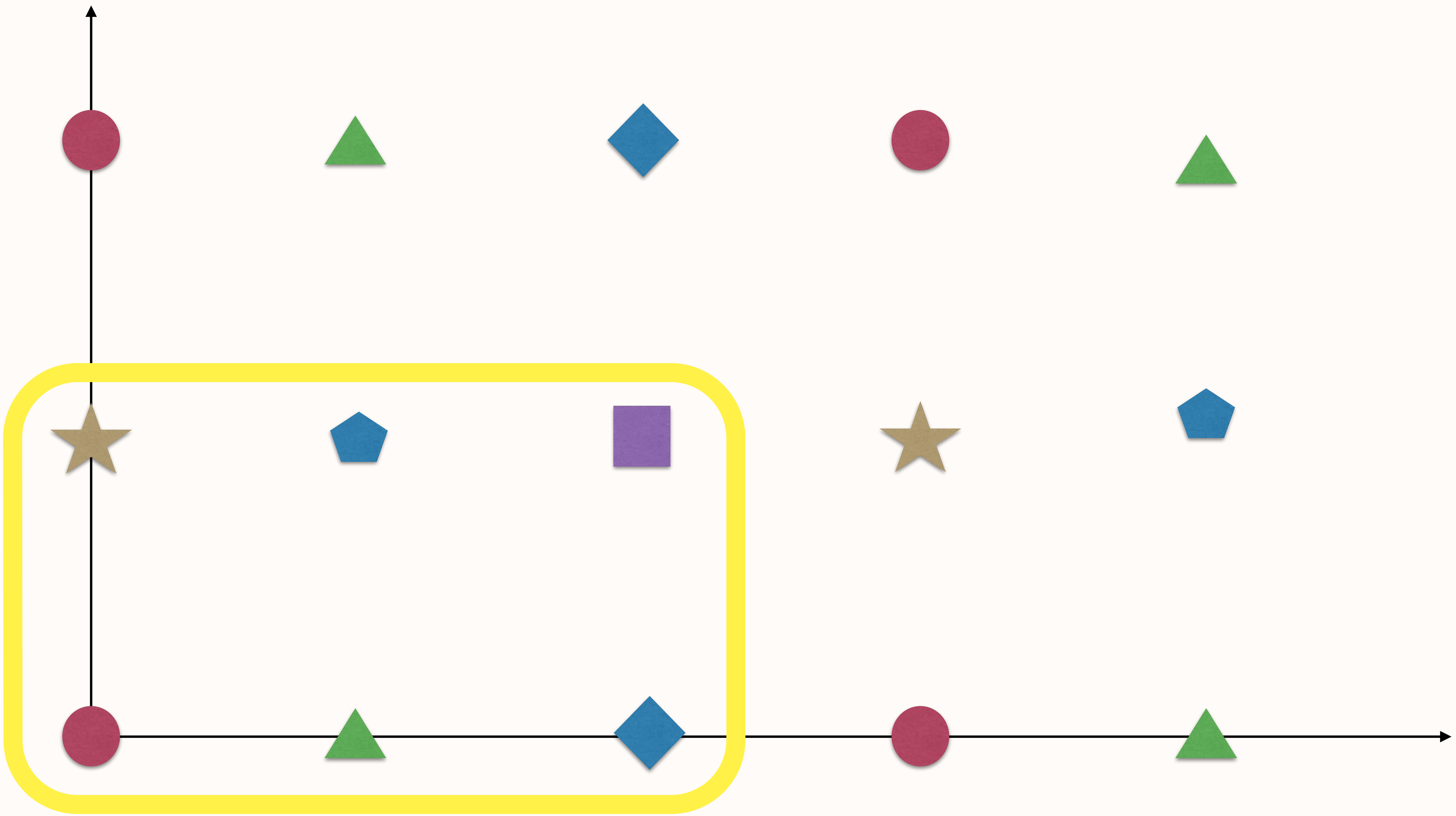

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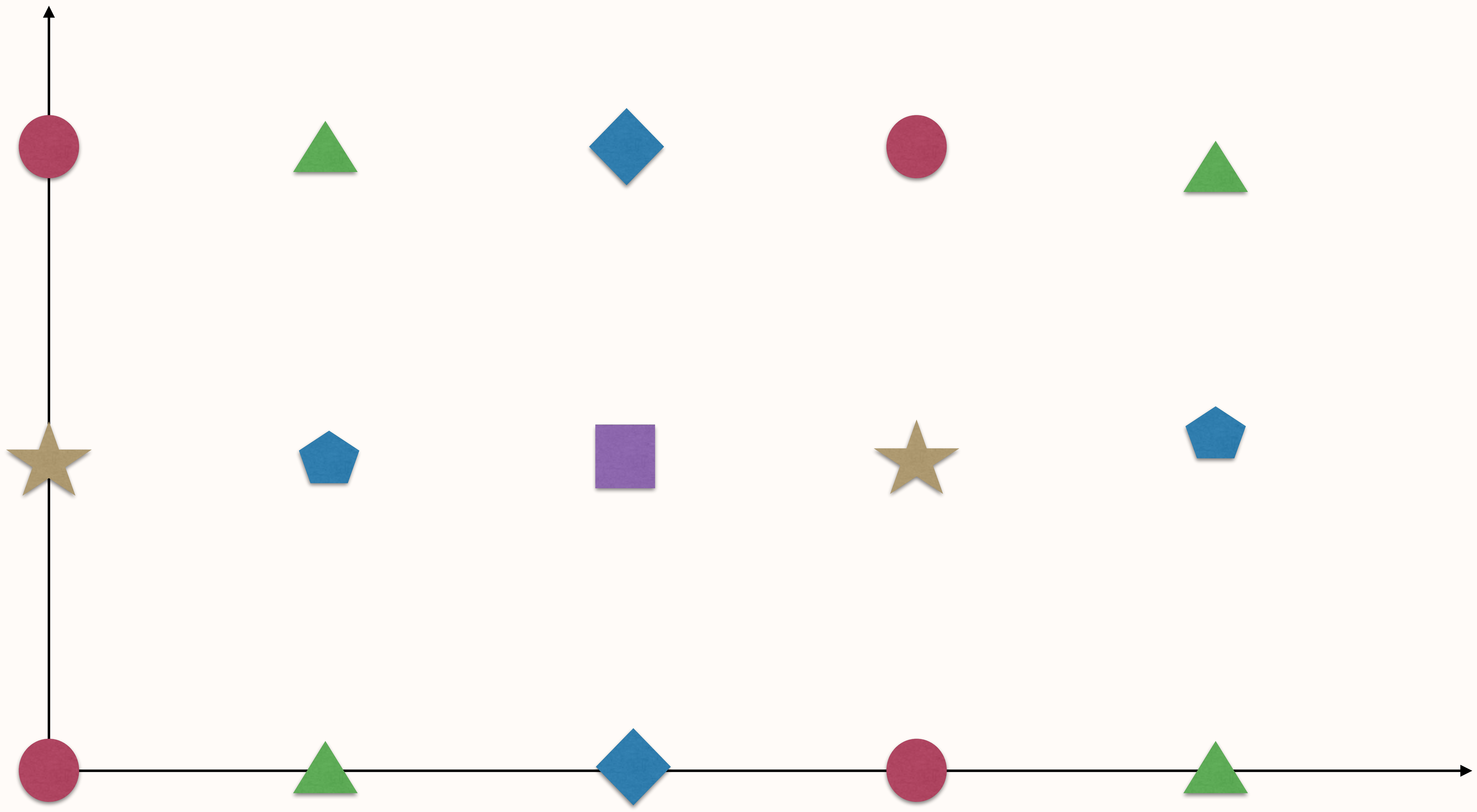

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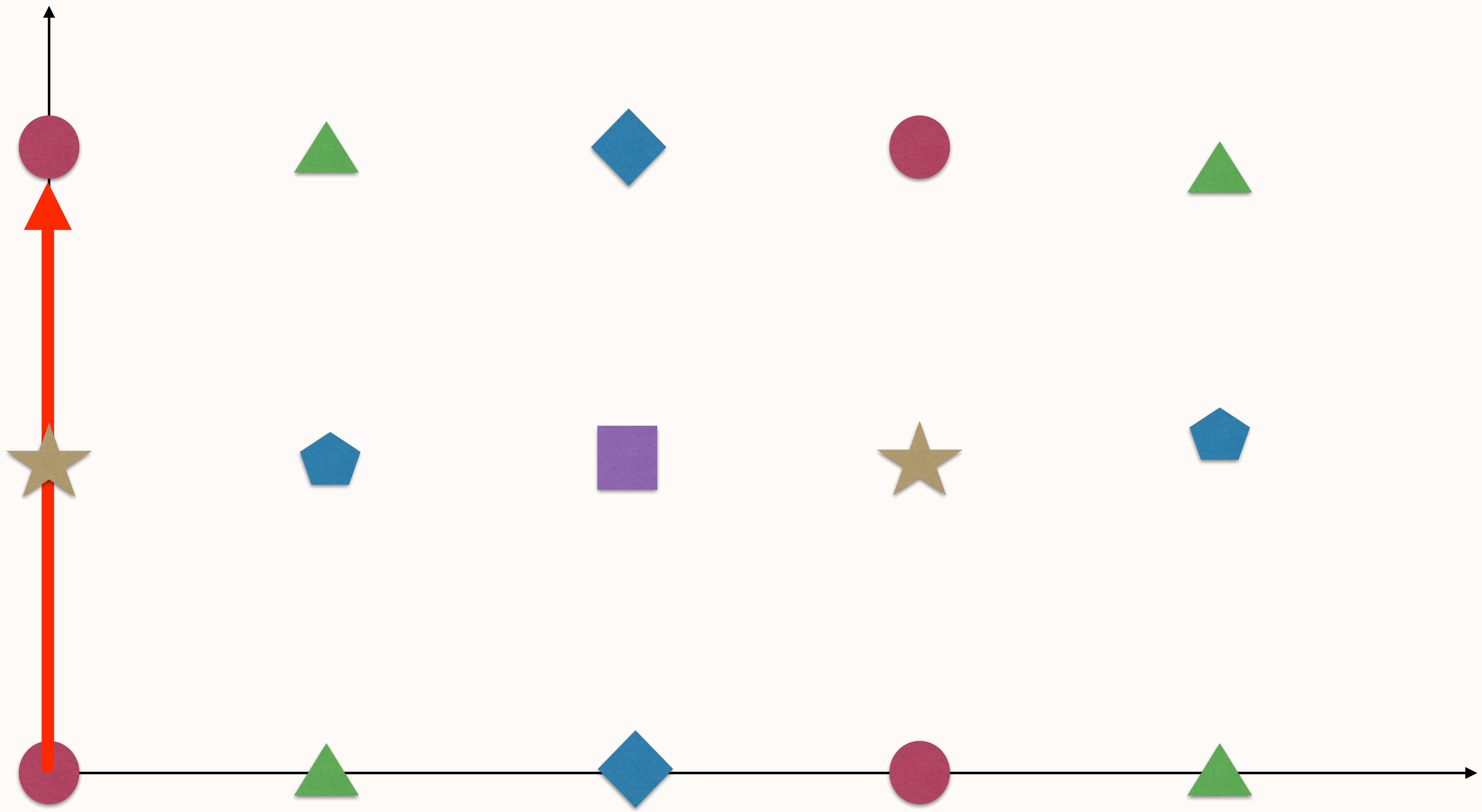

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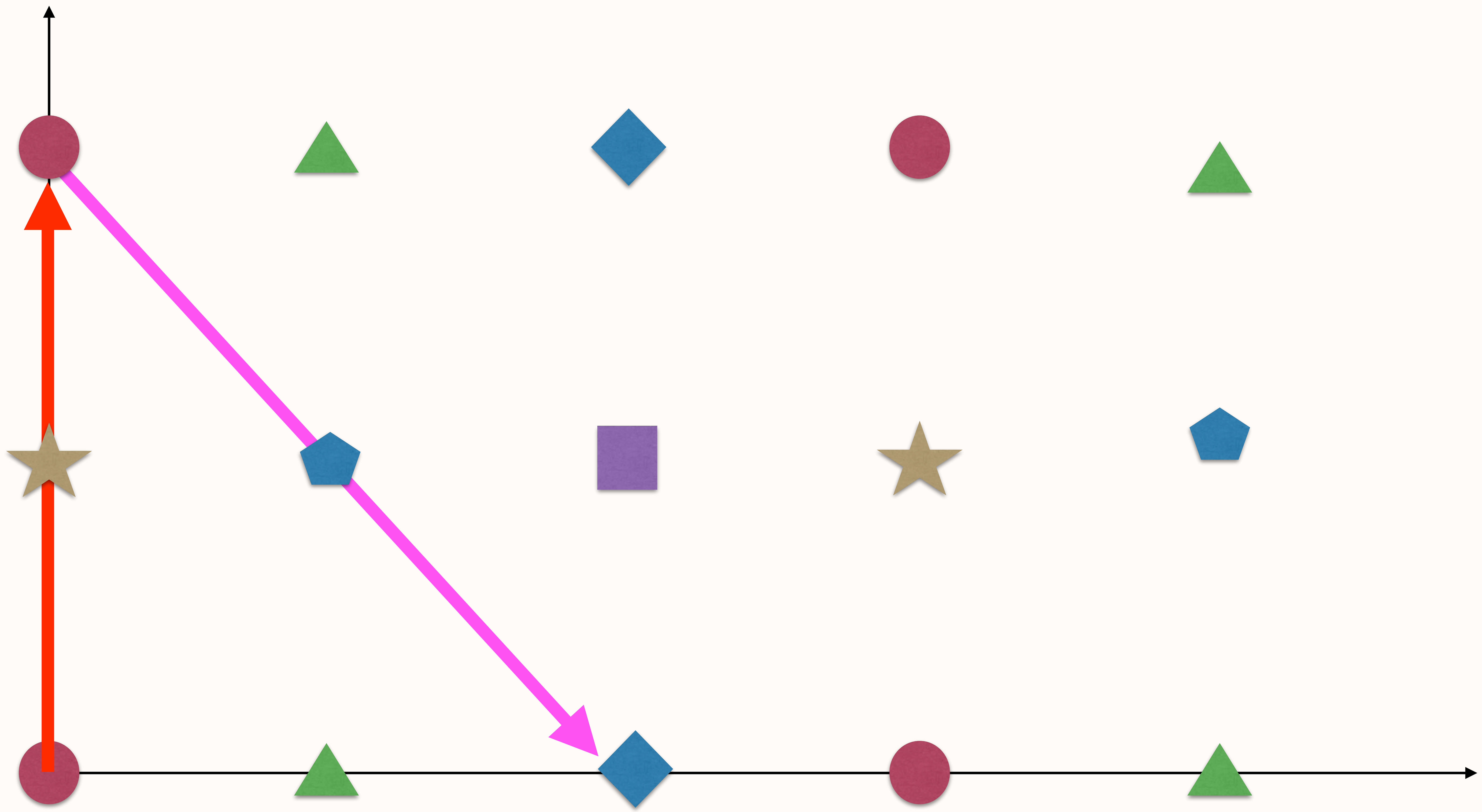

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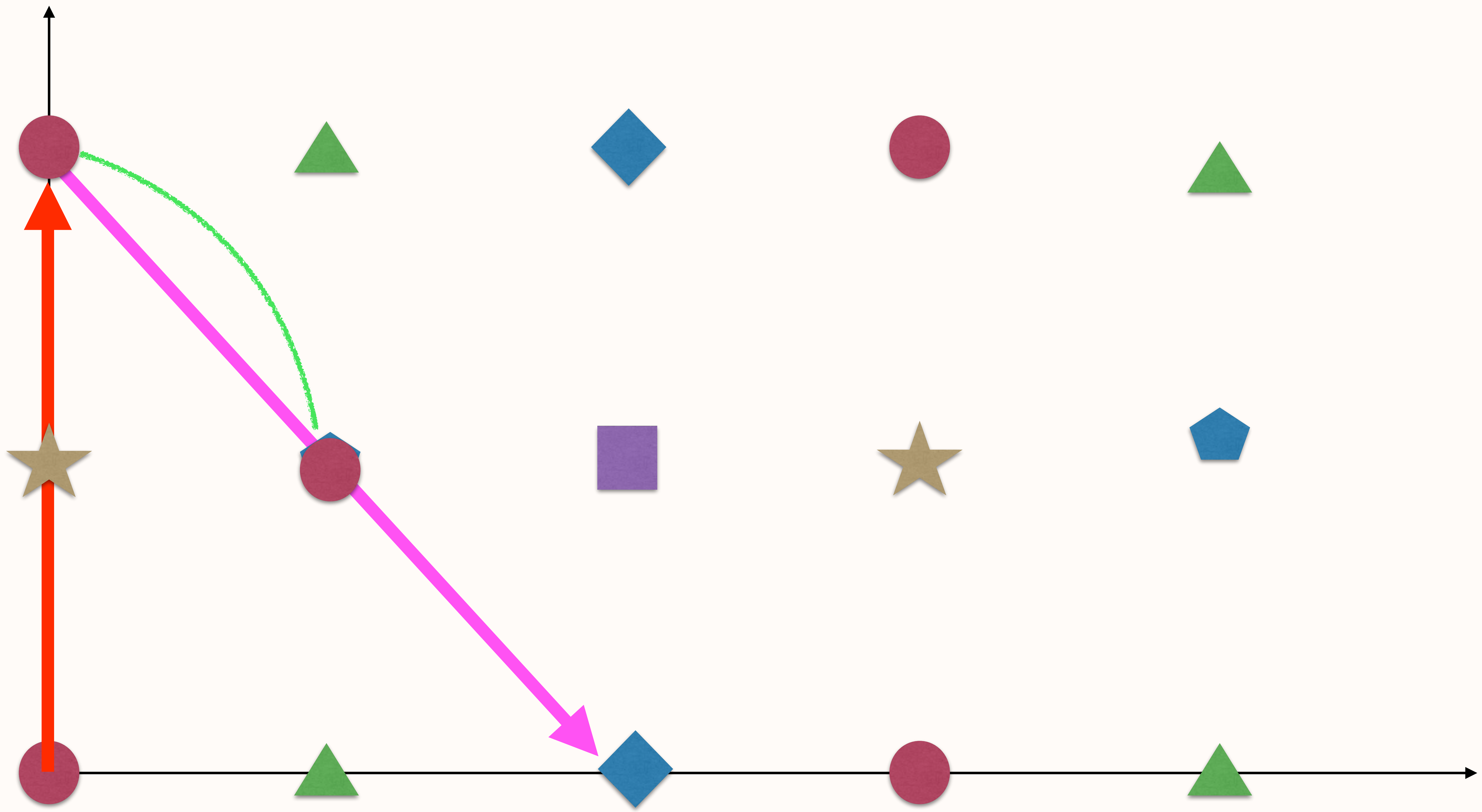

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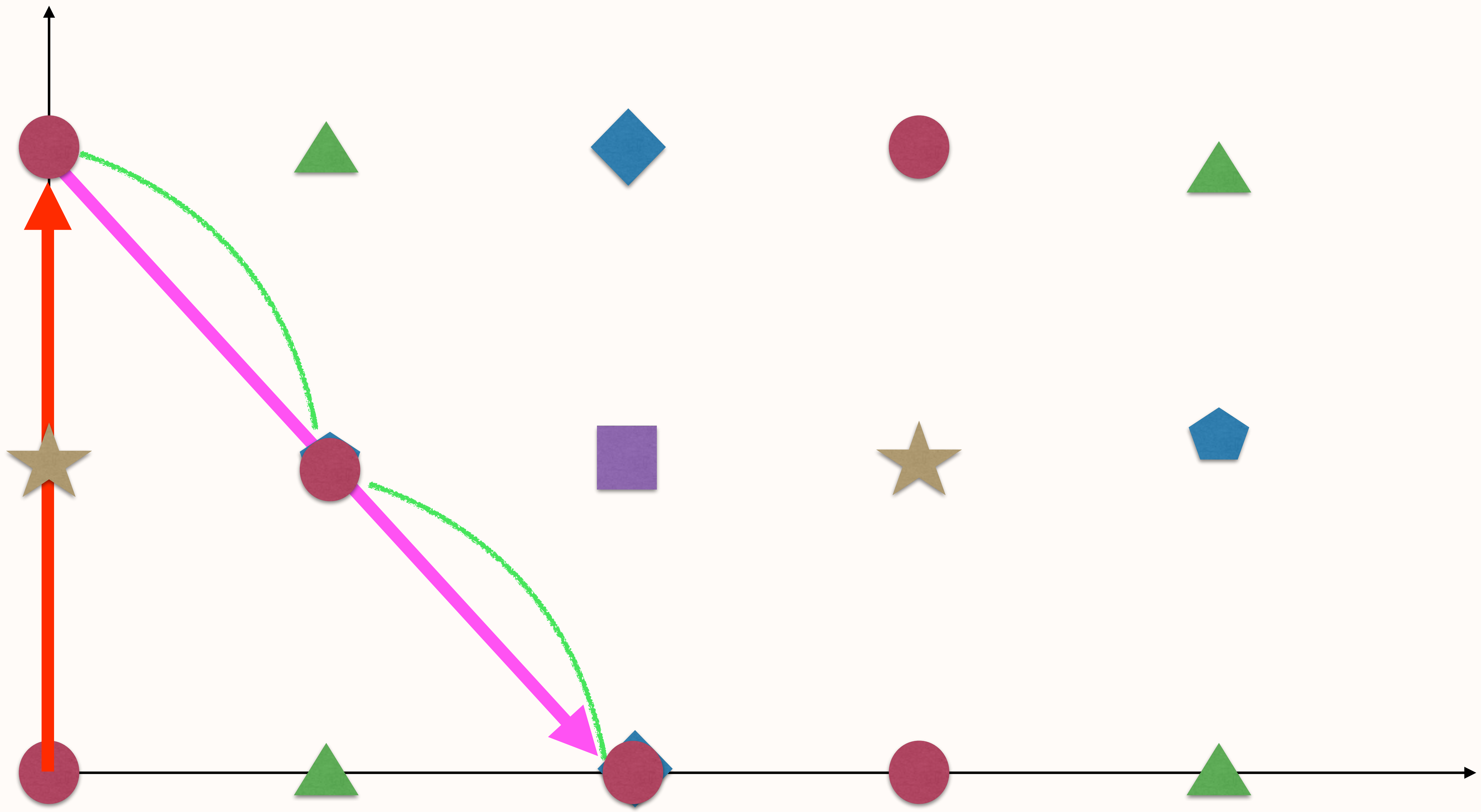

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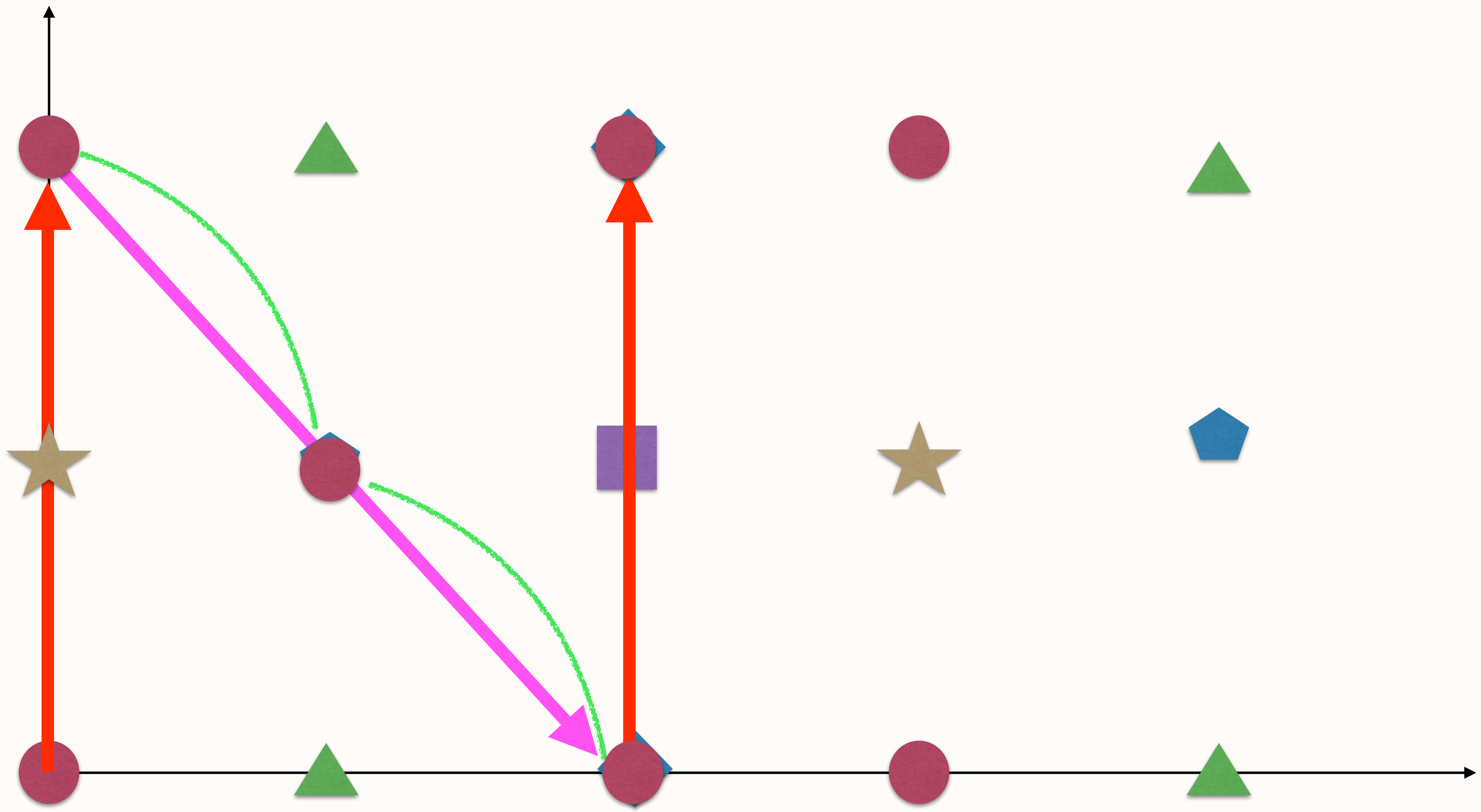

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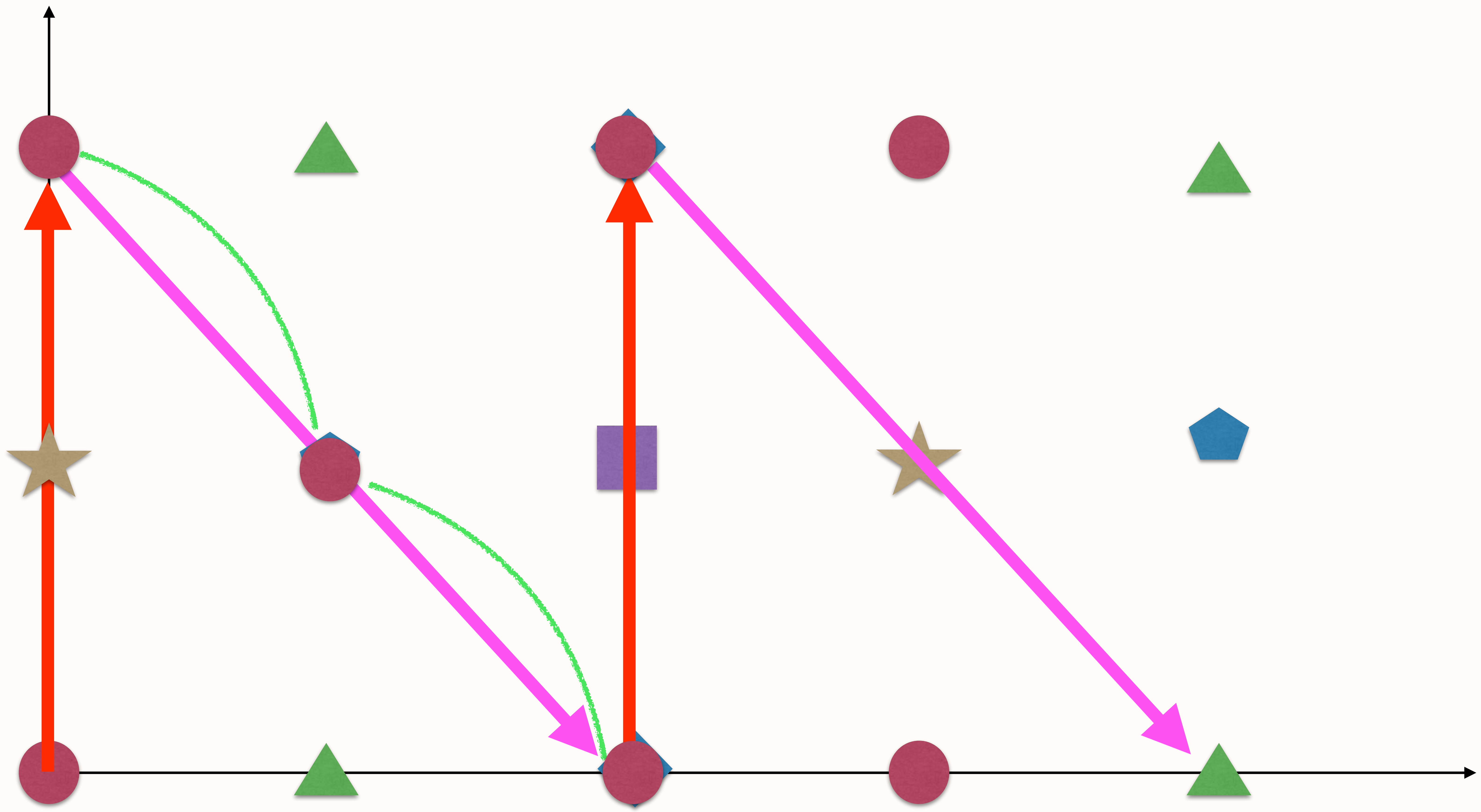

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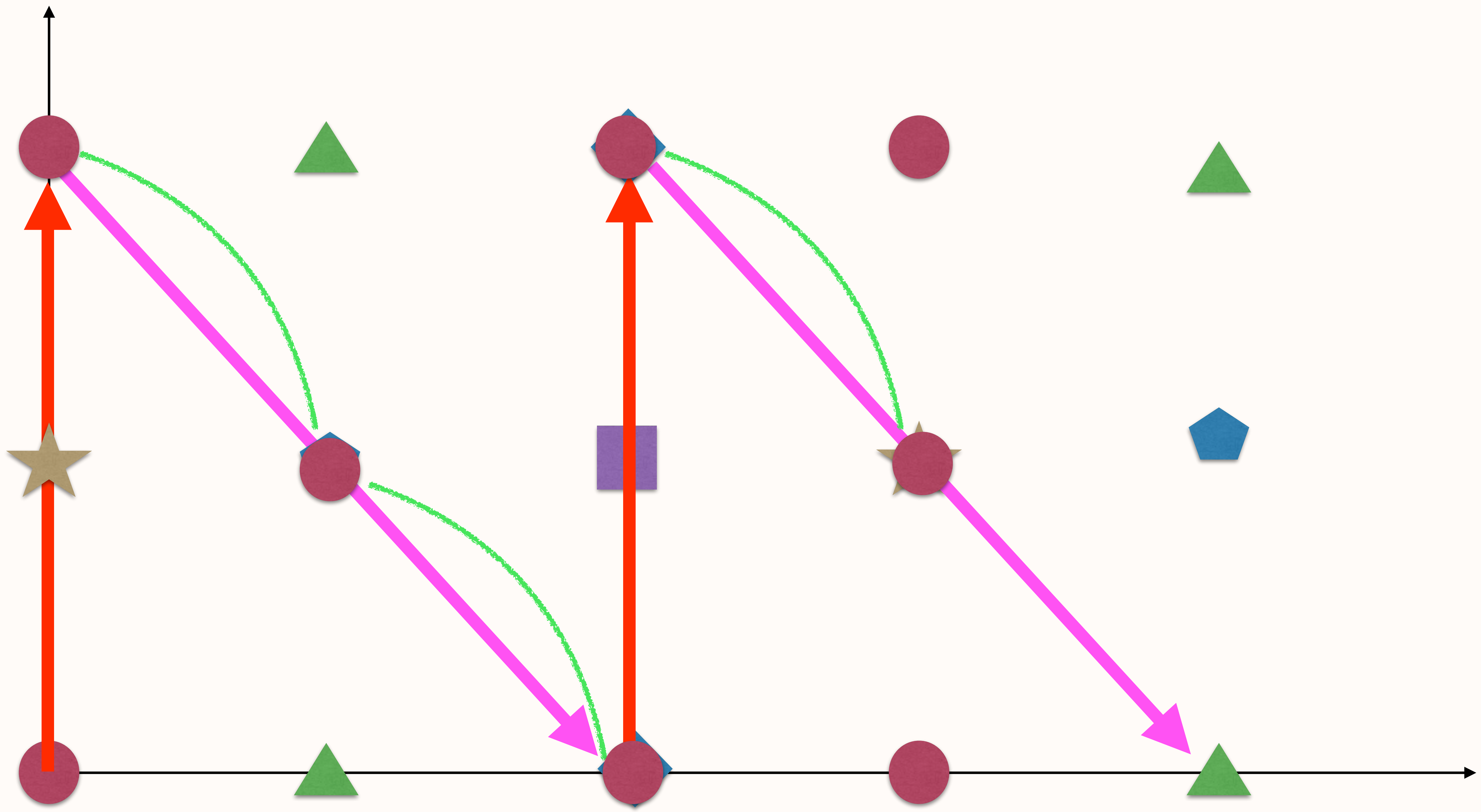

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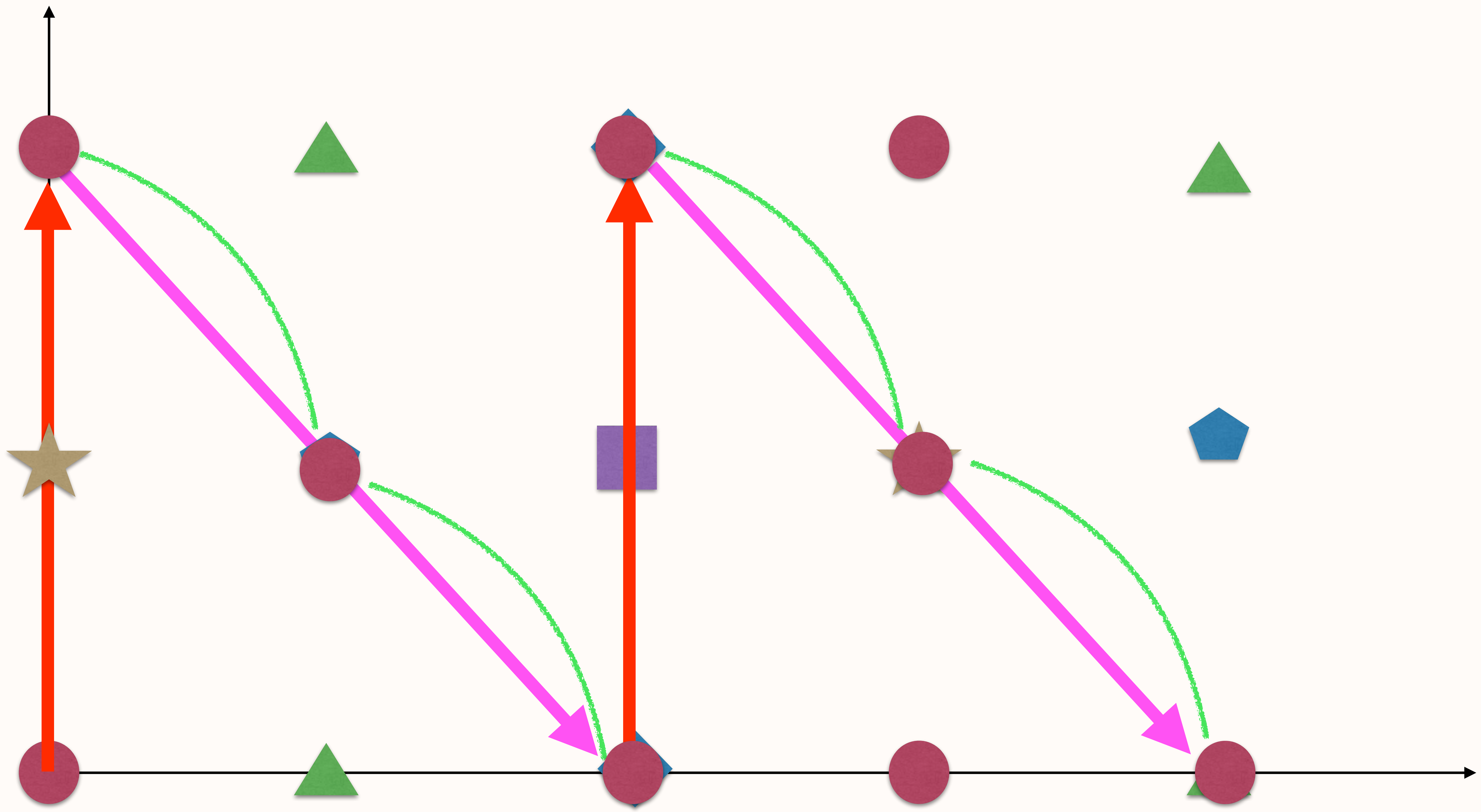

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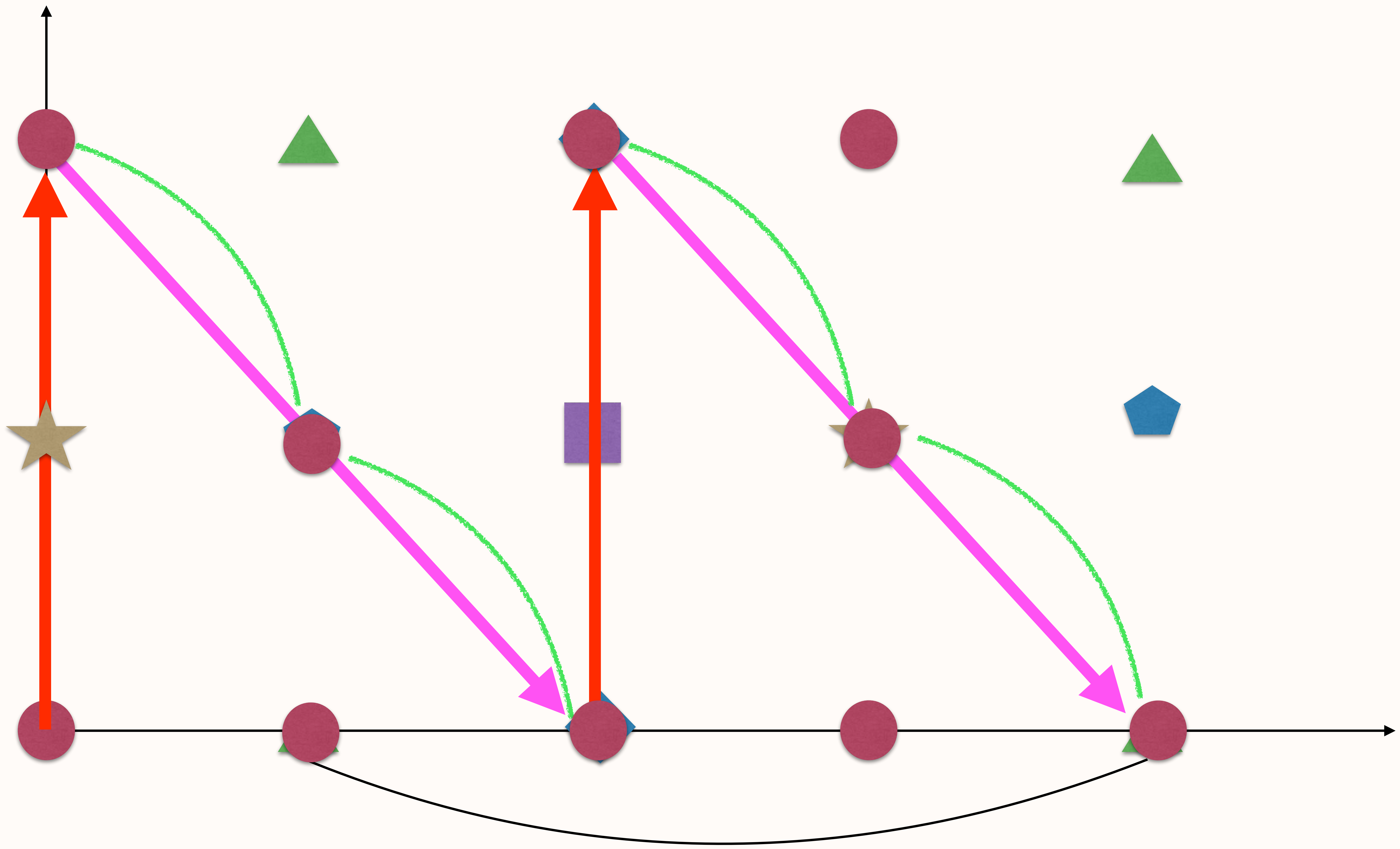
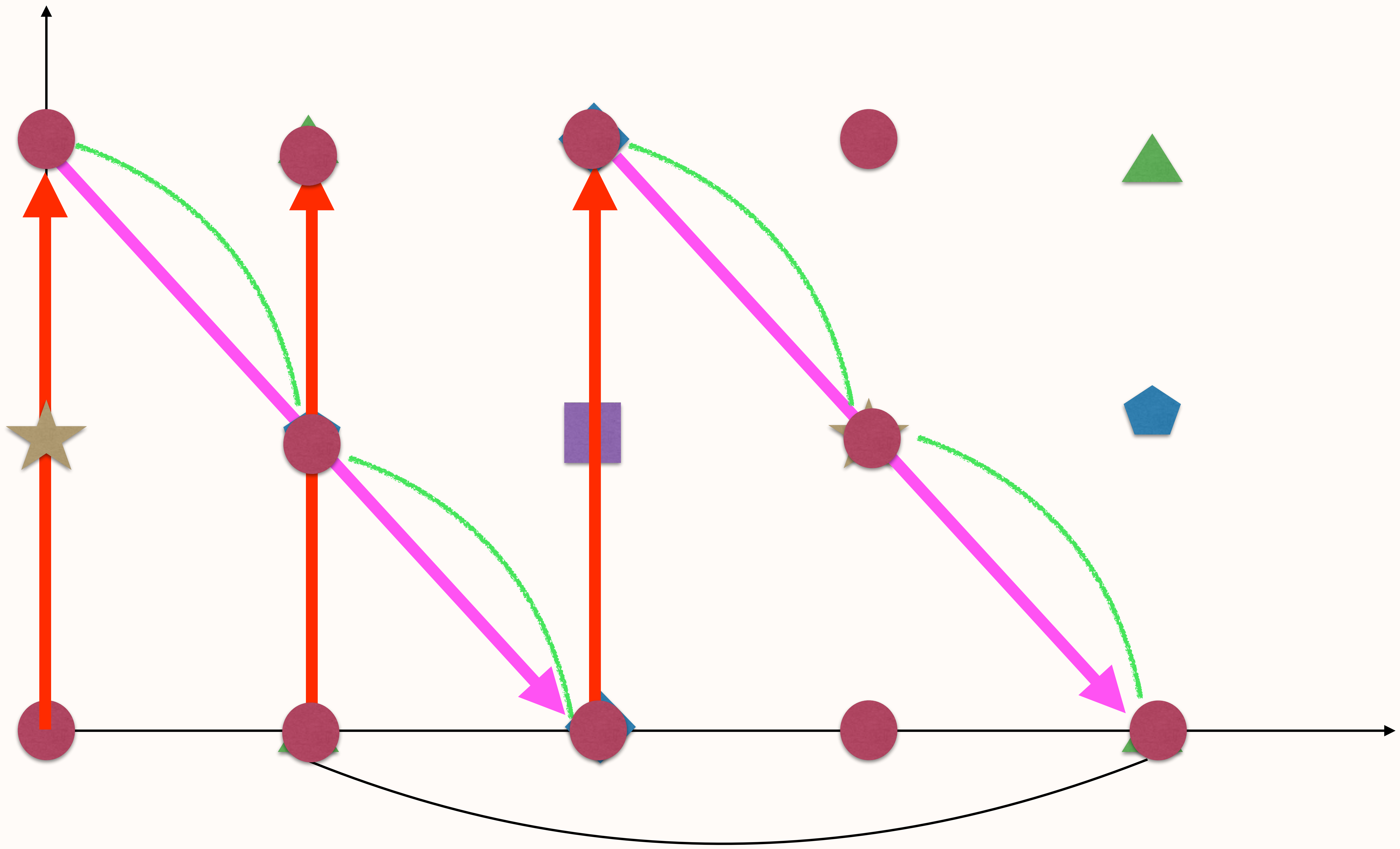


Torus identification

Choi-Kim, PRL55 (1985) 2637

with two confining forces



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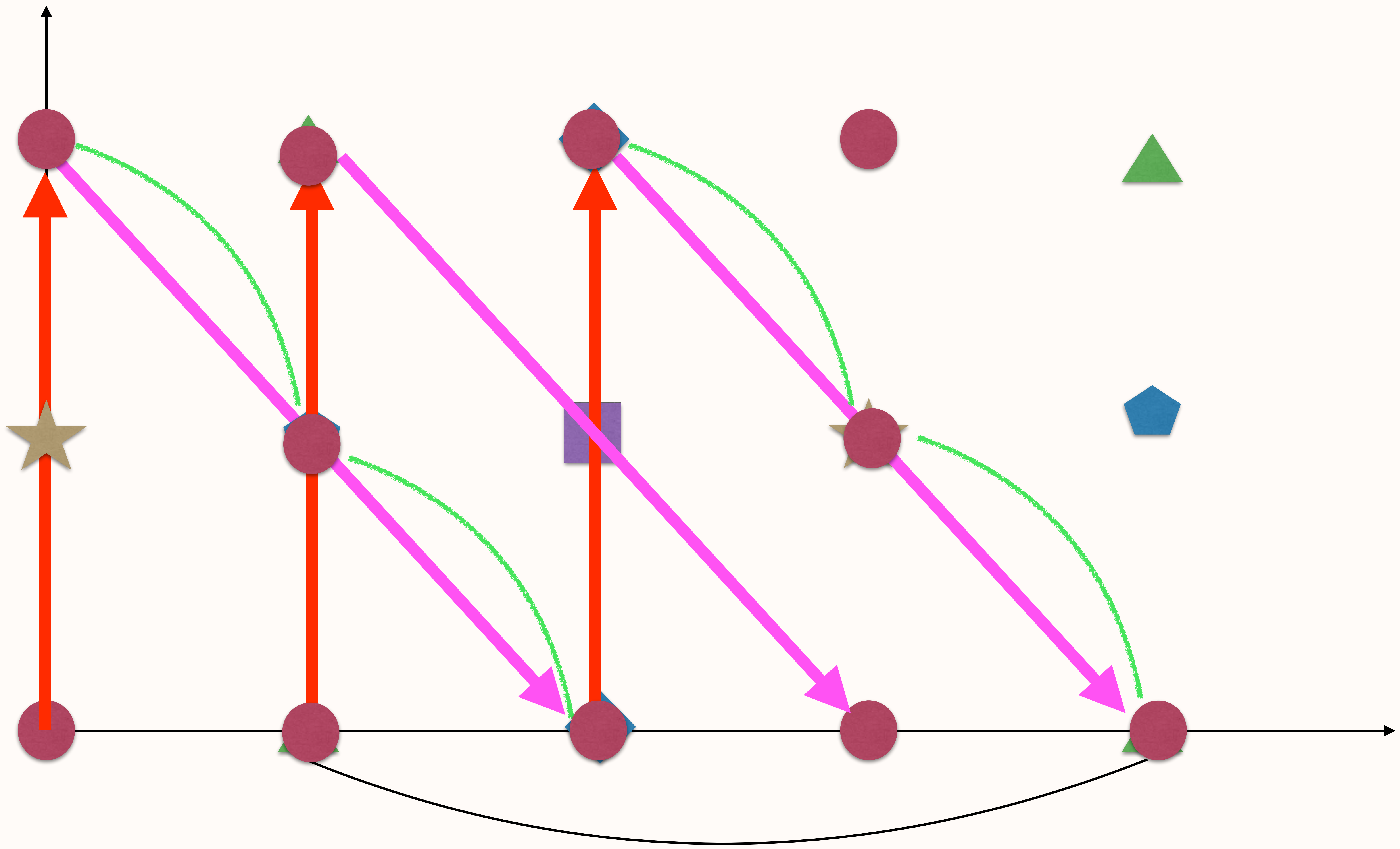

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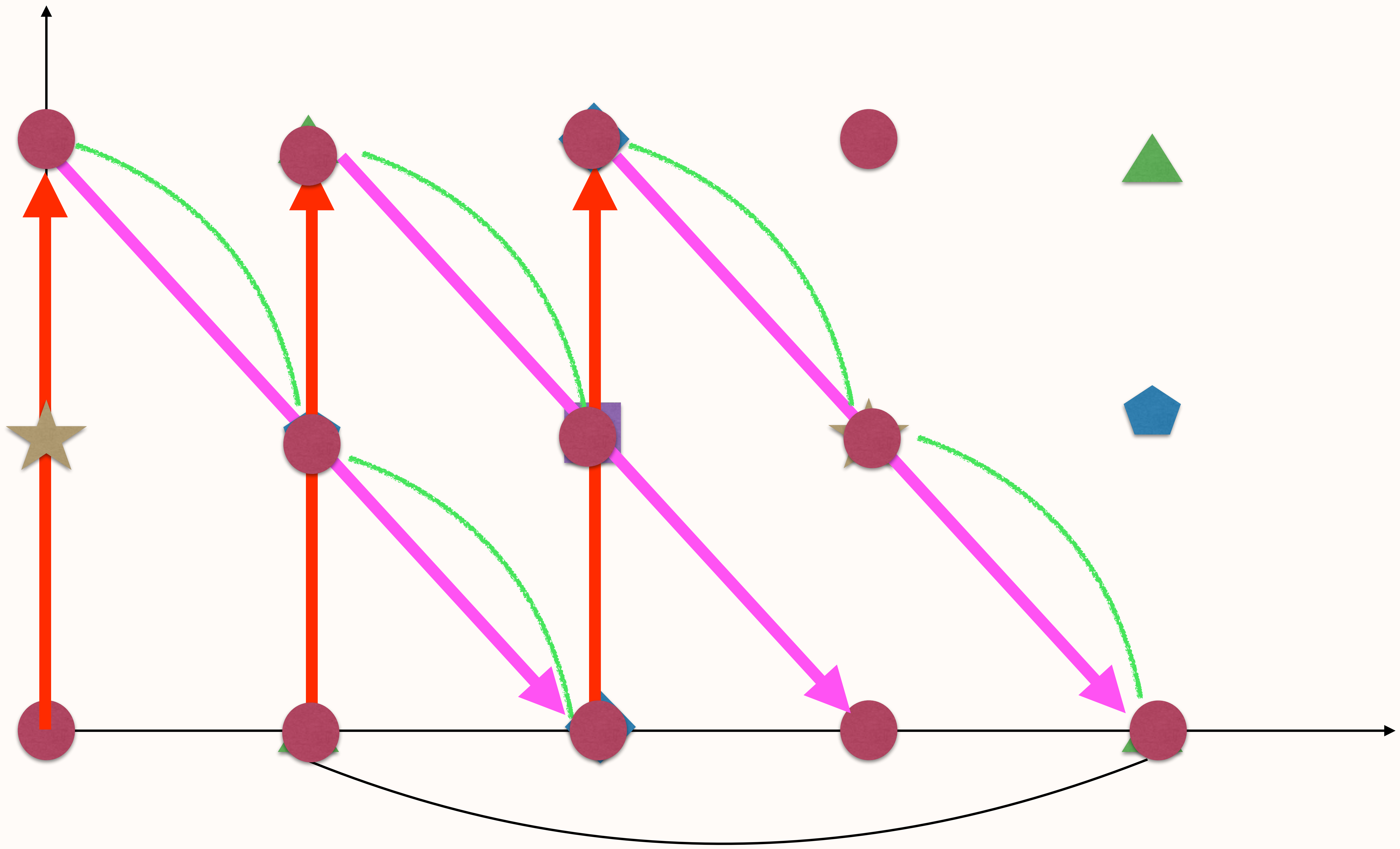

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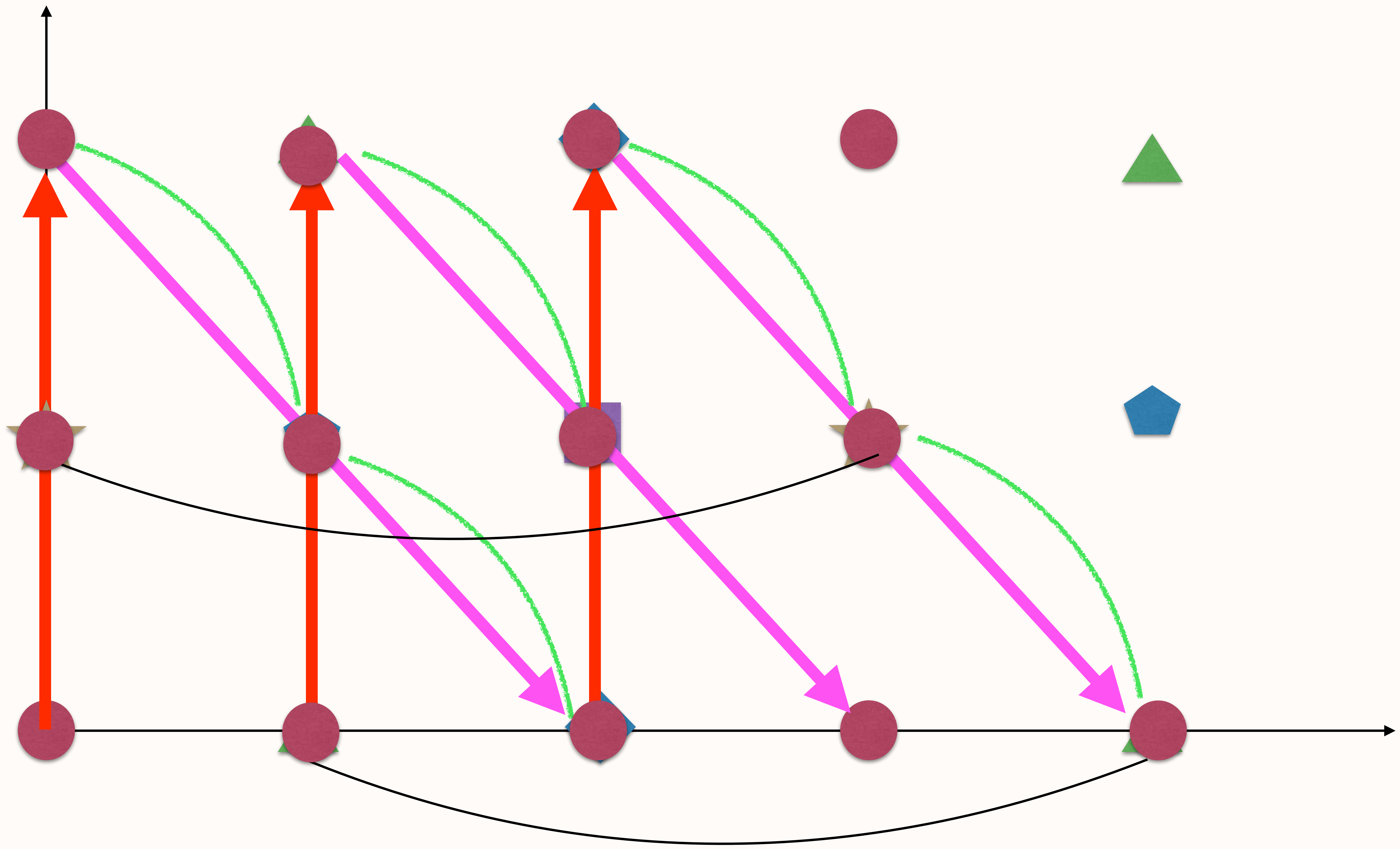

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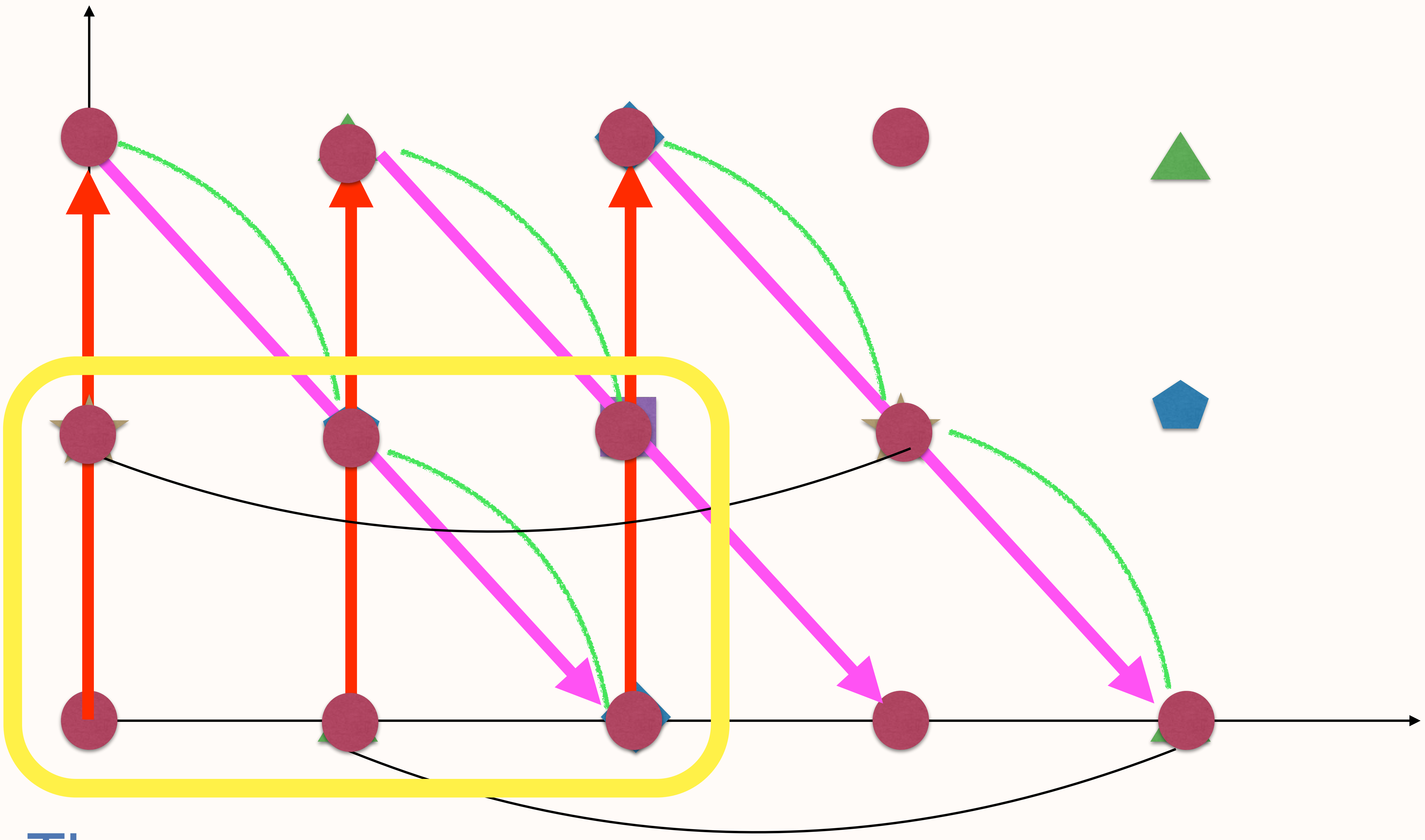
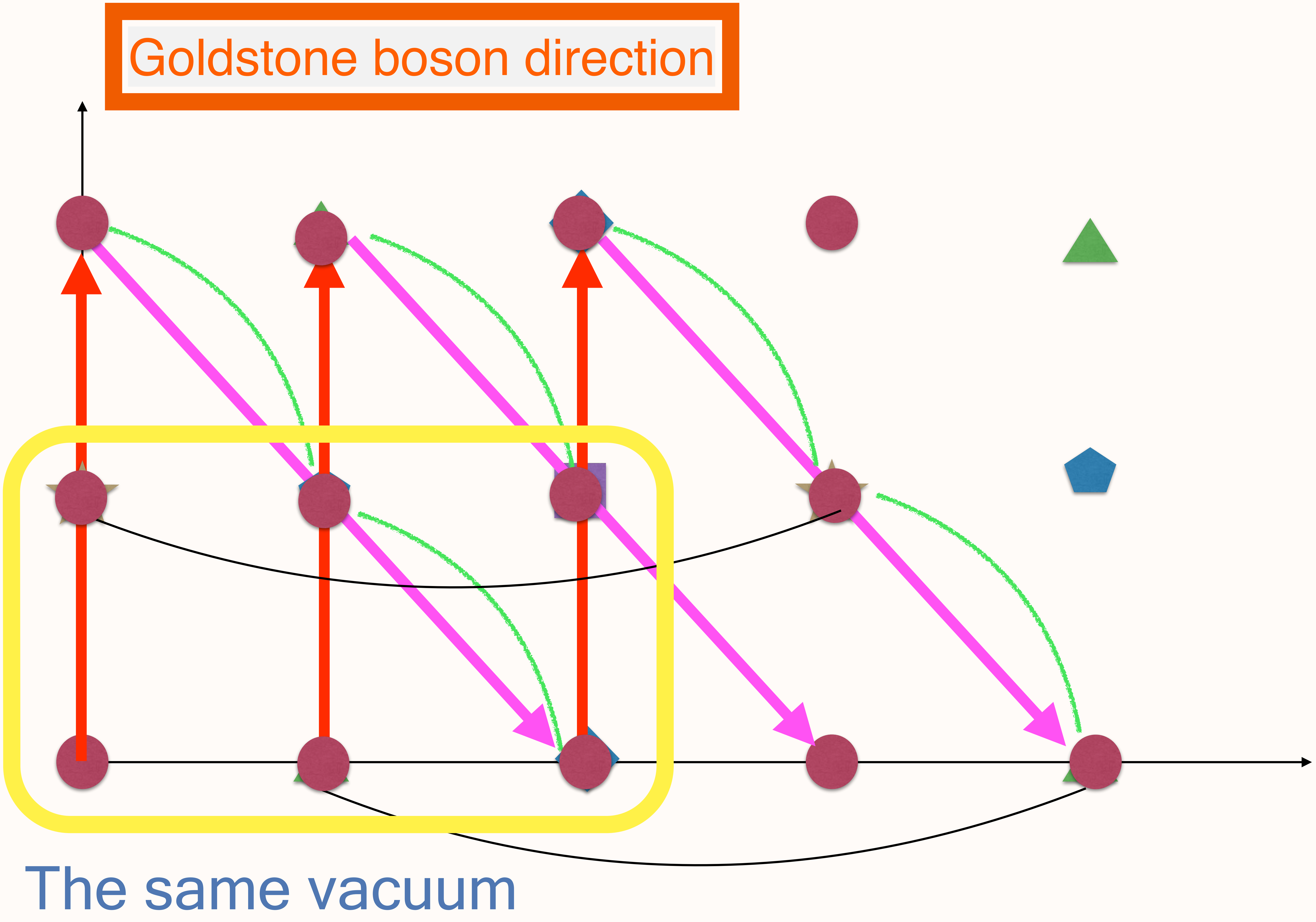
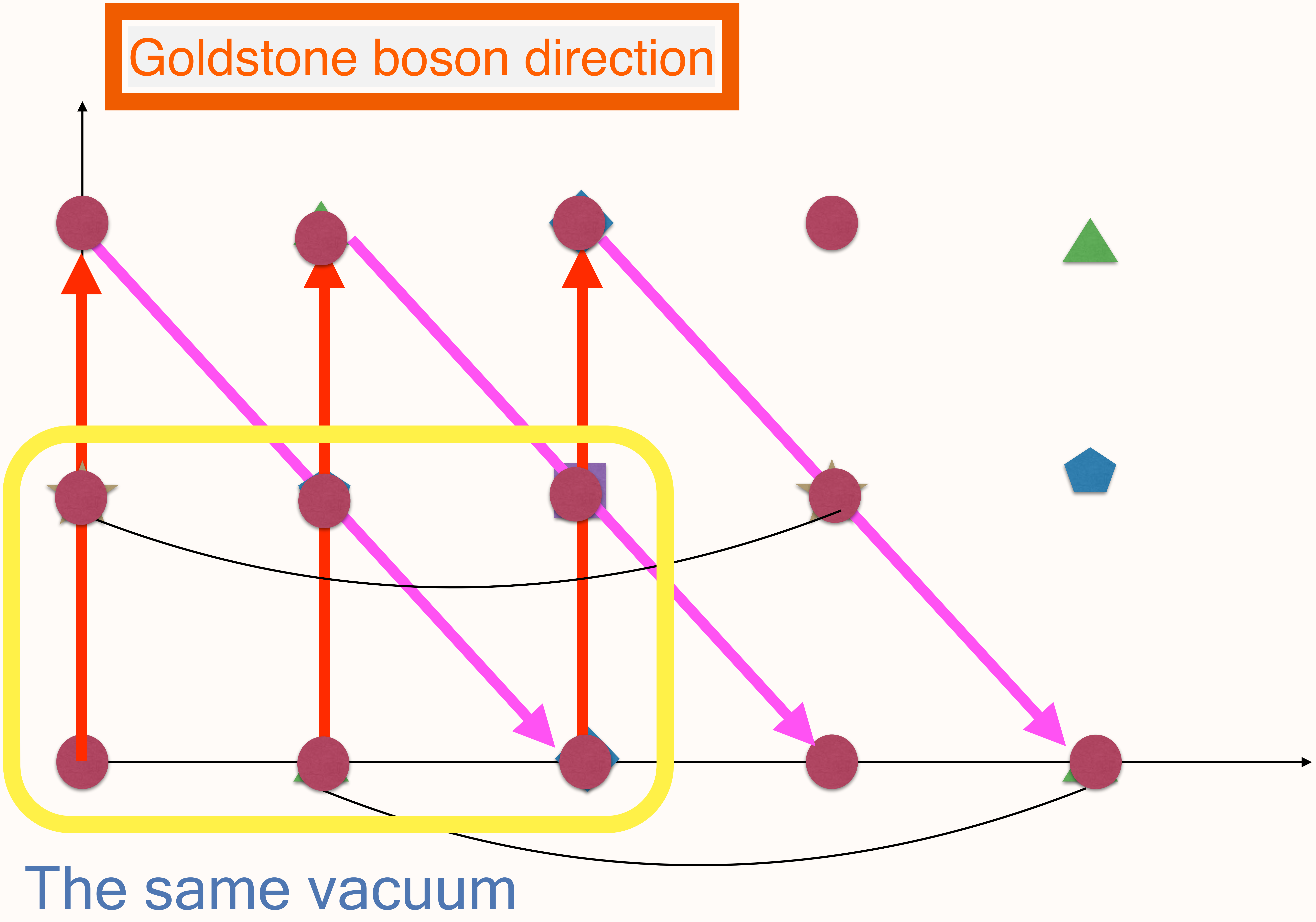


The same vacuum


Goldstone boson direction
The same vacuum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3. ’t Hooft mechanism

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If a gauge symmetry and a global symmetry are broken by one complex scalar by the BEHGHK mechanism, then the gauge symmetry is broken and a global symmetry remains unbroken.

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Q_{gauge}	Q_{global}
1	1

Unbroken $X=Q_{global}-Q_{gauge}$

$$\phi \to e^{i\alpha(x)Q_{\rm gauge}}e^{i\beta Q_{\rm global}}\phi$$

the α direction becomes the longitudinal mode of heavy gauge boson. The above transformation can be rewritten as

$$\phi \to e^{i(\alpha(x)+\beta)Q_{\rm gauge}}e^{i\beta(Q_{\rm global}-Q_{\rm gauge})}\phi$$

Redefining the local direction as $\alpha'(x) = \alpha(x) + \beta$, we obtain the transformation

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$$= \frac{g^2}{2} Q_a^2 v^2 (A_\mu - \frac{1}{gQ_a v}\partial^\mu a_\phi)^2$$

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$$= \frac{g^2}{2} Q_a^2 v^2 (A_\mu - \frac{1}{gQ_a v}\partial^\mu a_\phi)^2$$

So, the gauge boson becomes heavy and there remains the x-independent transformation parameter beta. The corresponding charge is a combination:

$X=Q_{global}-Q_{gauge}$

This process can be worked out at any step. When one global symmetry survives below a high energy scale, we consider another gauged U(1) and one more complex scalar to break two U(1)'s. Then, one global symmetry survives.

a_1 [= the phase of ϕ_1 (= $(V_1 + \rho_1)e^{ia_1/V_1})/\sqrt{2}$] are considered and only one Goldstone boson

$$\sqrt{M_{\rm MI}^2 + e^2V_1^2}\,(\cos\theta_G\, a_{\rm MI} - \sin\theta_G\, a_1)$$

re $\tan\theta_G = eV_1/M_{\rm MI}$. The orthogonal Goldestone boson direction

$$a' = \cos\theta_G\, a_1 + \sin\theta_G\, a_{\rm MI}$$

a global direction below the scale $\sqrt{M_{\rm MI}^2 + e^2V_1^2}$

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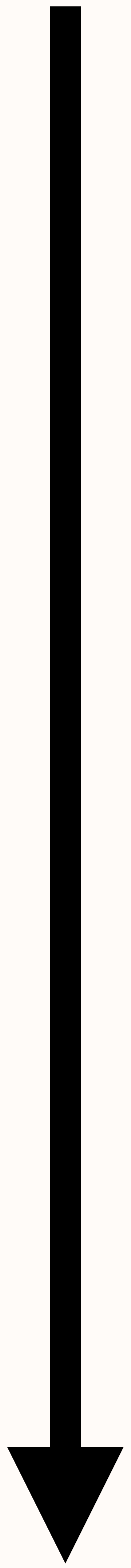
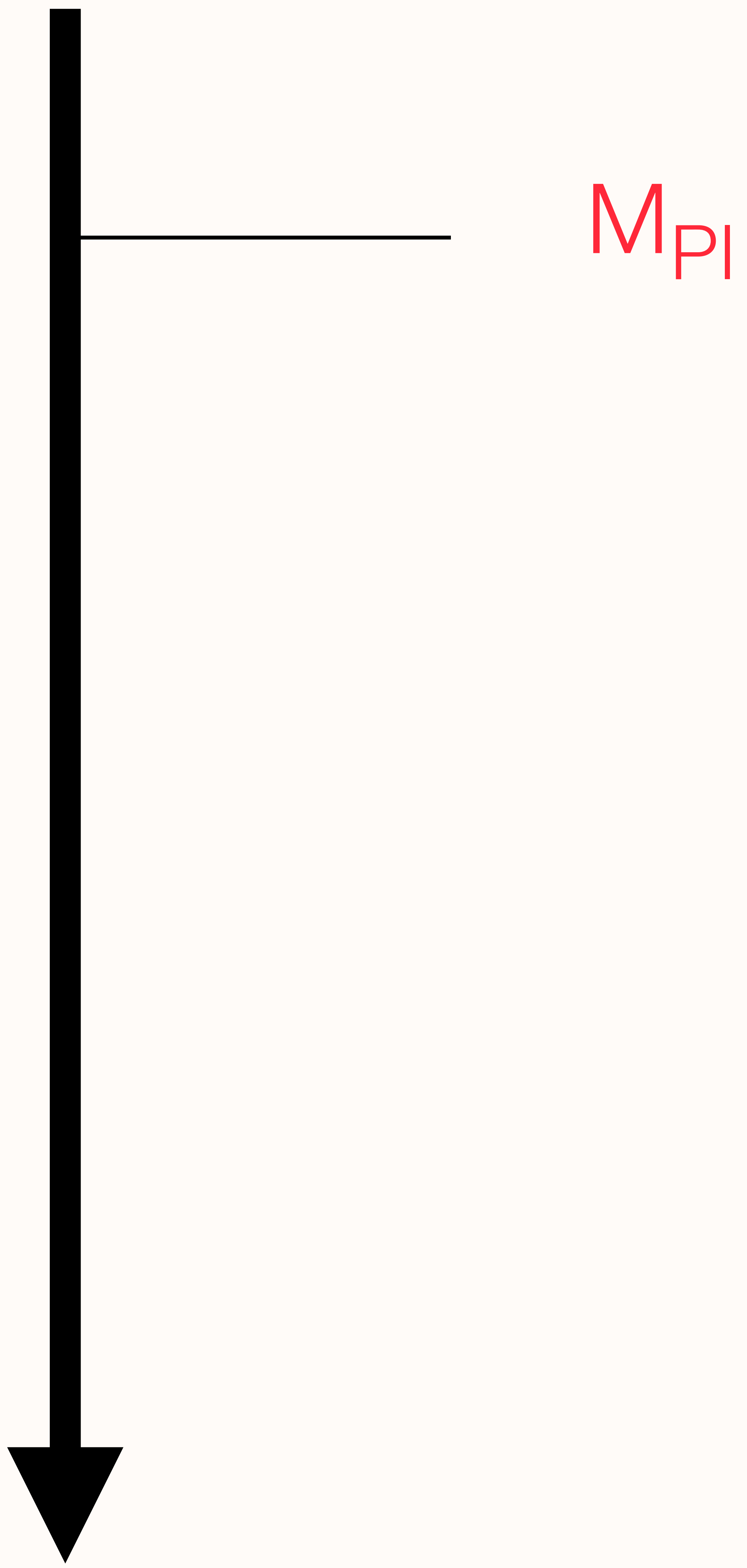
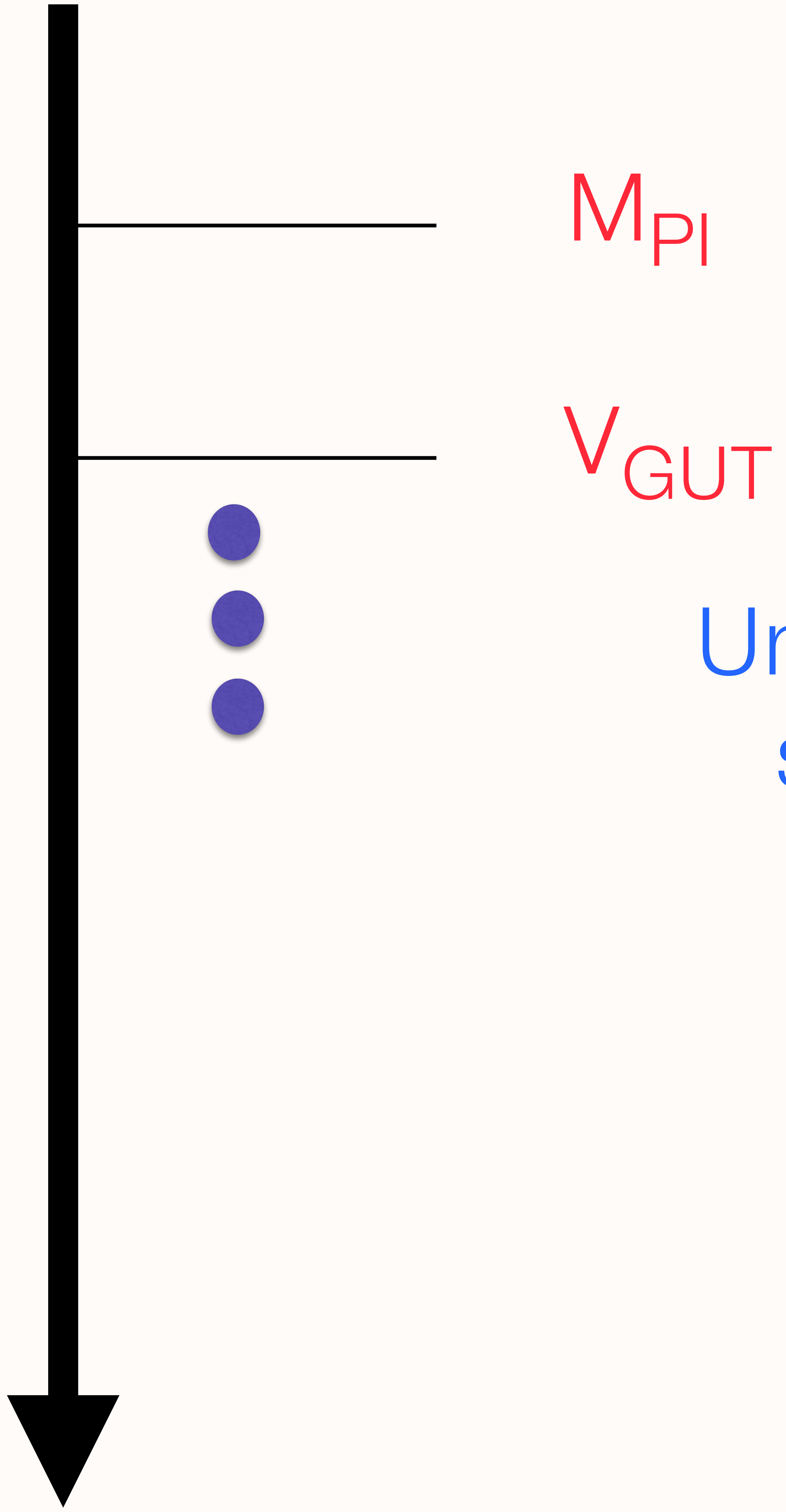
This process can be worked out further below the GUT scale as far as U(1) gauge symmetries (to be broken above the EW scale) are present. Then, one global symmetry survives down to the intermediate scale.



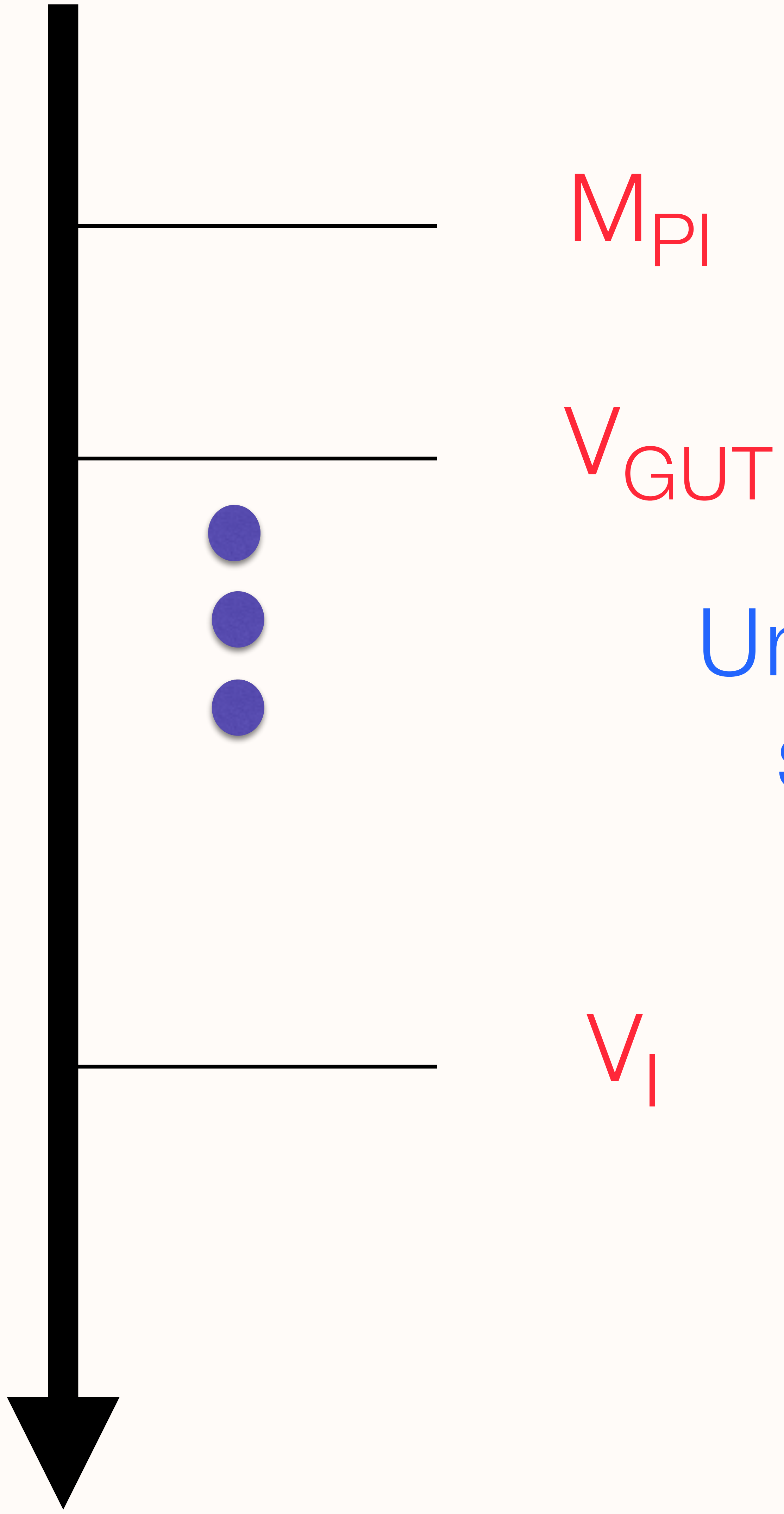


One complex scalar for one gauge symmetry breaking



One complex scalar for one gauge symmetry breaking

Until all wanted U(1) gauge symmetries are broken.



One complex scalar for one gauge symmetry breaking

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At the next step, a global symmetry is broken

4. Axion-photon-photon coupling

$$c^0_{a\gamma\gamma} = \frac{\mathrm{Tr}\,(Q_{\mathrm{em}})^2}{\mathrm{Tr}\,(T_3)^2}$$

One generator
on quark
fields, e.g. T_3

4. Axion-photon-photon coupling

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One generator on quark fields, e.g. T_3

If the quark representation is fundamental and only the SM quarks have PQ charges, then

Tr $|T_3|^2$=(1/2)x(number of chiral quarks). So,

$$c^0_{a\gamma\gamma} = \frac{\mathrm{Tr}\,(Q_{\rm em})^2}{\mathrm{Tr}\,(T_3)^2} = \frac{1}{\sin^2\theta_W}$$

$$c_{a\gamma\gamma} = c^0_{a\gamma\gamma} - (\text{Contribution from QCD chiral symmetry breaking})$$

We use 2 from m_u / m_d = 1/2

KSVZ: $Q_{\rm em}$	$c_{a\gamma\gamma}$
0	-2
$\pm\frac{1}{3}$	$-\frac{4}{3}$
$\pm\frac{2}{3}$	$\frac{2}{3}$
± 1	4
(m, m)	$-\frac{1}{3}$

DFSZ: $(q^c$-$e_L)$ pair	Higgs	$c_{a\gamma\gamma}$
non-SUSY (d^c, e)	H_d	$\frac{2}{3}$
non-SUSY (u^c, e)	H_u^*	$-\frac{4}{3}$
GUTs		$\frac{2}{3}$
SUSY		$\frac{2}{3}$

Phys. Rev. D55, 055006(1998)

String:	$c_{a\gamma\gamma}$	Comments
Ref. [25]	$-\frac{1}{3}$	Approximate
Ref. [26, 27]	$\frac{2}{3}$	Anom. U(1)

hep-ph/0612107

1405.6175; 1603.02145

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These numbers are usually marked as unification line, but it is not so.

hep-ph/0612107

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$m_u / m_d = 1/2$

In general, many quarks have PQ charges, and the tables on KSVZ and DFSZ do not make sense except in the experimental proposal for grants application. Further more vector like particles (must be of the KSVZ type) removed at the intermediate scale can contribute. Some ultraviolet completed theory is really prediction on the axion-photon-photon coupling.

One has to know all spectra with PQ, color, and EW charges.

1703.05345 and 1603.02145

Table 1 The $SU(5)\times U(1)_X$ states. Here, + represents helicity $+\frac{1}{2}$ and – represents helicity $-\frac{1}{2}$. Sum of $Q_{\rm anom}$ is multiplied by the index of the fundamental representation of $SU(3)_c$, $\frac{1}{2}$. The PQ symmetry, being chiral, counts quark and antiquark in the same way. The right-handed states in T_3 and T_5 are converted to the left-handed ones of T_9 and T_7, respectively. The bold entries are $Q_{\rm anom}/126$

Sect.	Colored states	$SU(5)_X$	Mult.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	Label	$Q_a^{\gamma\gamma}$
U	$(\underline{+++--};--+)(0^8)'$	$\overline{\mathbf{10}}_{-1}$		-6	-6	$+6$	0	0	0	$-1638(\mathbf{-13})$	C_2	-3276
U	$(\underline{+----};+--)(0^8)'$	$\mathbf{5}_{+3}$		$+6$	-6	-6	0	0	0	$-126(\mathbf{-1})$	C_1	-294
T_4^0	$(\underline{+----};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^8)'$	$\mathbf{5}_{+3}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_3$	-882
T_4^0	$(\underline{+++--};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^8)'$	$\overline{\mathbf{10}}_{-1}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_4$	-756
T_4^0	$(\underline{10000};\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	$\mathbf{5}_{-2}$	2	$+4$	$+4$	$+4$	0	0	0	$+756(\mathbf{+6})$	$2C_5$	$+1008$
T_4^0	$(\underline{-10000};\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	$\overline{\mathbf{5}}_{+2}$	2	$+4$	$+4$	$+4$	0	0	0	$+756(\mathbf{+6})$	$2C_6$	$+1008$
T_6^0	$(\underline{10000};000)(0^5;\frac{-1}{2}\,\frac{+1}{2}\,0)'$	$\mathbf{5}_{-2}$	3	0	0	0	-12	0	0	0	$3C_7$	0
T_6^0	$(\underline{-10000};000)(0^5;\frac{+1}{2}\,\frac{-1}{2}\,0)'$	$\overline{\mathbf{5}}_{+2}$	3	0	0	0	$+12$	0	0	0	$3C_8$	0
T_7^0	$(\underline{-10000};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\overline{\mathbf{5}}_{+2}$	1	-2	-2	-2	0	$+9$	$+3$	$-972(-\frac{54}{7})$	C_9	-1296
T_7^0	$(\underline{+10000};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\mathbf{5}_{-2}$	1	-2	-2	-2	0	$+9$	$+3$	$-972(-\frac{54}{7})$	C_{10}	-1296
T_3^0	$(\underline{+++--};000)(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\overline{\mathbf{10}}_{-1}$	1	0	0	0	0	$+9$	$+3$	$-594(-\frac{33}{7})$	C_{11}	-1188
T_9^0	$(\underline{++---};000)(0^5;\frac{+1}{4}\,\frac{+1}{4}\,\frac{-2}{4})'$	$\mathbf{10}_{+1}$	1	0	0	0	0	-9	-3	$+594(+\frac{33}{7})$	C_{12}	$+1188$
				-16	-28	$+8$	0	$+18$	$+6$	-3492		-5406

1703.05345 and 1603.02145

These are family q.n.

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U	$(\underline{+----};+--)(0^8)'$	$\mathbf{5}_{+3}$		$+6$	-6	-6	0	0	0	$-126(\mathbf{-1})$	C_1	-294
T_4^0	$(\underline{+----};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^8)'$	$\mathbf{5}_{+3}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_3$	-882
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T_4^0	$(\underline{10000};\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	$\mathbf{5}_{-2}$	2	$+4$	$+4$	$+4$	0	0	0	$+756(\mathbf{+6})$	$2C_5$	$+1008$
T_4^0	$(\underline{-10000};\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	$\overline{\mathbf{5}}_{+2}$	2	$+4$	$+4$	$+4$	0	0	0	$+756(\mathbf{+6})$	$2C_6$	$+1008$
T_6^0	$(\underline{10000};000)(0^5;\frac{-1}{2}\,\frac{+1}{2}\,0)'$	$\mathbf{5}_{-2}$	3	0	0	0	-12	0	0	0	$3C_7$	0
T_6^0	$(\underline{-10000};000)(0^5;\frac{+1}{2}\,\frac{-1}{2}\,0)'$	$\overline{\mathbf{5}}_{+2}$	3	0	0	0	$+12$	0	0	0	$3C_8$	0
T_7^0	$(\underline{-10000};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\overline{\mathbf{5}}_{+2}$	1	-2	-2	-2	0	$+9$	$+3$	$-972(-\frac{54}{7})$	C_9	-1296
T_7^0	$(\underline{+10000};\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\mathbf{5}_{-2}$	1	-2	-2	-2	0	$+9$	$+3$	$-972(-\frac{54}{7})$	C_{10}	-1296
T_3^0	$(\underline{+++--};000)(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\overline{\mathbf{10}}_{-1}$	1	0	0	0	0	$+9$	$+3$	$-594(-\frac{33}{7})$	C_{11}	-1188
T_9^0	$(\underline{++---};000)(0^5;\frac{+1}{4}\,\frac{+1}{4}\,\frac{-2}{4})'$	$\mathbf{10}_{+1}$	1	0	0	0	0	-9	-3	$+594(+\frac{33}{7})$	C_{12}	$+1188$
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U	$(+----;+--)(0^8)'$	$\mathbf{5}_{+3}$		+6	−6	−6	0	0	0	−126(**−1**)	C_1	−294
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T_4^0	$(10000;\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	$\mathbf{5}_{-2}$	2	+4	+4	+4	0	0	0	+756(**+6**)	$2C_5$	+1008
T_4^0	$(-10000;\frac{1}{3}\,\frac{1}{3}\,\frac{1}{3})(0^8)'$	$\overline{\mathbf{5}}_{+2}$	2	+4	+4	+4	0	0	0	+756(**+6**)	$2C_6$	+1008
T_6^0	$(10000;000)(0^5;\frac{-1}{2}\,\frac{+1}{2}\,0)'$	$\mathbf{5}_{-2}$	3	0	0	0	−12	0	0	0	$3C_7$	0
T_6^0	$(-10000;000)(0^5;\frac{+1}{2}\,\frac{-1}{2}\,0)'$	$\overline{\mathbf{5}}_{+2}$	3	0	0	0	+12	0	0	0	$3C_8$	0
T_7^0	$(-10000;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\overline{\mathbf{5}}_{+2}$	1	−2	−2	−2	0	+9	+3	$-972(-\frac{54}{7})$	C_9	−1296
T_7^0	$(+10000;\frac{-1}{6}\,\frac{-1}{6}\,\frac{-1}{6})(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\mathbf{5}_{-2}$	1	−2	−2	−2	0	+9	+3	$-972(-\frac{54}{7})$	C_{10}	−1296
T_3^0	$(+++--;000)(0^5;\frac{-1}{4}\,\frac{-1}{4}\,\frac{+2}{4})'$	$\overline{\mathbf{10}}_{-1}$	1	0	0	0	0	+9	+3	$-594(-\frac{33}{7})$	C_{11}	−1188
T_9^0	$(++---;000)(0^5;\frac{+1}{4}\,\frac{+1}{4}\,\frac{-2}{4})'$	$\mathbf{10}_{+1}$	1	0	0	0	0	−9	−3	$+594(+\frac{33}{7})$	C_{12}	+1188
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Two families from T4 and one family from U

1703.05345 and 1603.02145

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T_4^0	$(+----;\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0^8)'$	$\mathbf{5}_{+3}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_3$	-882
T_4^0	$(+++--;\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0^8)'$	$\overline{\mathbf{10}}_{-1}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_4$	-756
T_4^0	$(10000;\frac{1}{3}\frac{1}{3}\frac{1}{3})(0^8)'$	$\mathbf{5}_{-2}$	2	$+4$	$+4$	$+4$	0	0	0	$+756(\mathbf{+6})$	$2C_5$	$+1008$
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T_6^0	$(10000;000)(0^5;\frac{-1}{2}\frac{+1}{2}0)'$	$\mathbf{5}_{-2}$	3	0	0	0	-12	0	0	0	$3C_7$	0
T_6^0	$(-10000;000)(0^5;\frac{+1}{2}\frac{-1}{2}0)'$	$\overline{\mathbf{5}}_{+2}$	3	0	0	0	$+12$	0	0	0	$3C_8$	0
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T_9^0	$(++---;000)(0^5;\frac{+1}{4}\frac{+1}{4}\frac{-2}{4})'$	$\mathbf{10}_{+1}$	1	0	0	0	0	-9	-3	$+594(+\frac{33}{7})$	C_{12}	$+1188$
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T_4^0	$(\underline{+++--};\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0^8)'$	$\overline{\mathbf{10}}_{-1}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_4$	-756
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T_6^0	$(\underline{10000};000)(0^5;\frac{-1}{2}\frac{+1}{2}0)'$	$\mathbf{5}_{-2}$	3	0	0	0	-12	0	0	0	$3C_7$	0
T_6^0	$(\underline{-10000};000)(0^5;\frac{+1}{2}\frac{-1}{2}0)'$	$\overline{\mathbf{5}}_{+2}$	3	0	0	0	$+12$	0	0	0	$3C_8$	0
T_7^0	$(\underline{-10000};\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0^5;\frac{-1}{4}\frac{-1}{4}\frac{+2}{4})'$	$\overline{\mathbf{5}}_{+2}$	1	-2	-2	-2	0	$+9$	$+3$	$-972(-\frac{54}{7})$	C_9	-1296
T_7^0	$(\underline{+10000};\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0^5;\frac{-1}{4}\frac{-1}{4}\frac{+2}{4})'$	$\mathbf{5}_{-2}$	1	-2	-2	-2	0	$+9$	$+3$	$-972(-\frac{54}{7})$	C_{10}	-1296
T_3^0	$(\underline{+++--};000)(0^5;\frac{-1}{4}\frac{-1}{4}\frac{+2}{4})'$	$\overline{\mathbf{10}}_{-1}$	1	0	0	0	0	$+9$	$+3$	$-594(-\frac{33}{7})$	C_{11}	-1188
T_9^0	$(\underline{++---};000)(0^5;\frac{+1}{4}\frac{+1}{4}\frac{-2}{4})'$	$\mathbf{10}_{+1}$	1	0	0	0	0	-9	-3	$+594(+\frac{33}{7})$	C_{12}	$+1188$
				-16	-28	$+8$	0	$+18$	$+6$	-3492		-5406

Two families from T4 and one family from U

1703.05345 and 1603.02145

These are family q.n.

Table 1 The $SU(5)\times U(1)_X$ states. Here, + represents helicity $+\frac{1}{2}$ and – represents helicity $-\frac{1}{2}$. Sum of $Q_{\rm anom}$ is multiplied by the index of the fundamental representation of $SU(3)_c$, $\frac{1}{2}$. The PQ symmetry, being chiral, counts quark and antiquark in the same way. The right-handed states in T_3 and T_5 are converted to the left-handed ones of T_9 and T_7, respectively. The bold entries are $Q_{\rm anom}/126$

Sect.	Colored states	$SU(5)_X$	Mult.	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$Q_{\rm anom}$	Label	$Q_a^{\gamma\gamma}$
U	$(\underline{+++--};--+)(0^8)'$	$\overline{\bf 10}_{-1}$		-6	-6	$+6$	0	0	0	$-1638(\mathbf{-13})$	C_2	-3276
U	$(\underline{+----};+--)(0^8)'$	${\bf 5}_{+3}$		$+6$	-6	-6	0	0	0	$-126(\mathbf{-1})$	C_1	-294
T_4^0	$(\underline{+----};\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0^8)'$	${\bf 5}_{+3}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_3$	-882
T_4^0	$(\underline{+++--};\frac{-1}{6}\frac{-1}{6}\frac{-1}{6})(0^8)'$	$\overline{\bf 10}_{-1}$	2	-2	-2	-2	0	0	0	$-378(\mathbf{-3})$	$2C_4$	-756
T_4^0	$(\underline{10000};\frac{1}{3}\frac{1}{3}\frac{1}{3})(0^8)'$	${\bf 5}_{-2}$	2	$+4$	$+4$	$+4$	0	0	0	$+756(\mathbf{+6})$	$2C_5$	$+1008$
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Adding contributions from other tables, -9312

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T_6^0	$(\underline{-10000};000)(0^5;\frac{+1}{2}\frac{-1}{2}0)'$	$\overline{\mathbf{5}}_{+2}$	3	0	0	0	$+12$	0	0	0	$3C_8$	0
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Two families from T4 and one family from U

$$c_{a\gamma\gamma} \simeq \frac{-9312}{-3492} - 2 = \frac{2}{3}$$

Adding contributions from other tables, -9312

The unification value

5. Model-independent axion in string theory

U(1)global


U(1)global
Z2


TOE

Openning of string theory by the GS term

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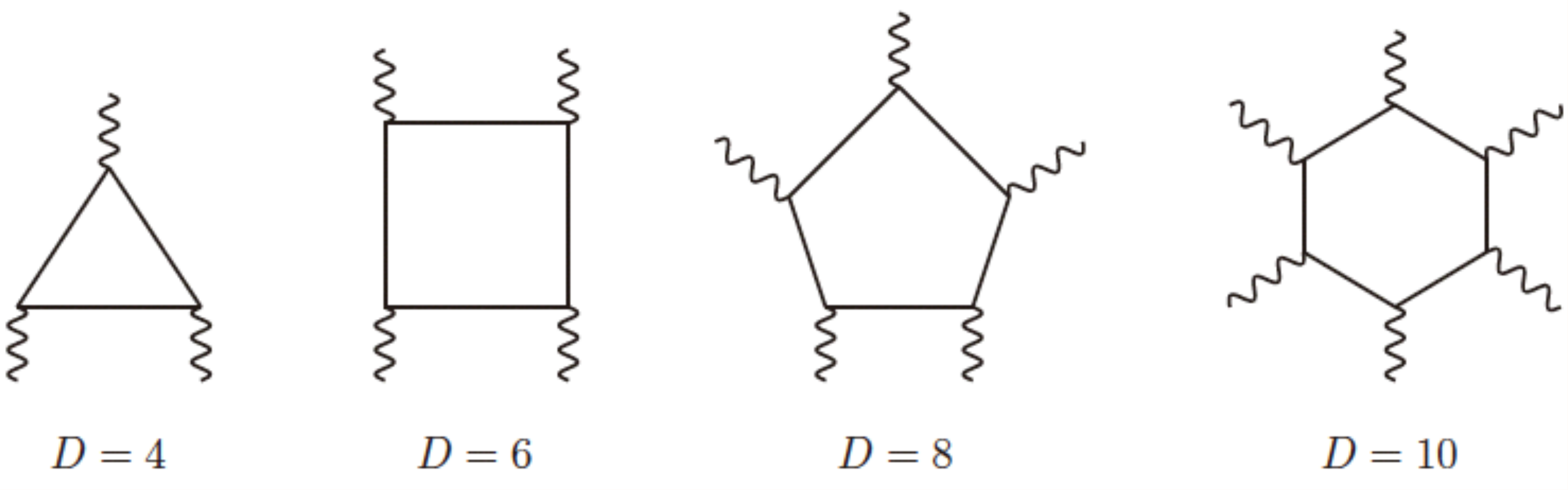
One needs a term (GS-term) to cancel the gauge and gravitational anomalies.

Anomalies: even dimensions



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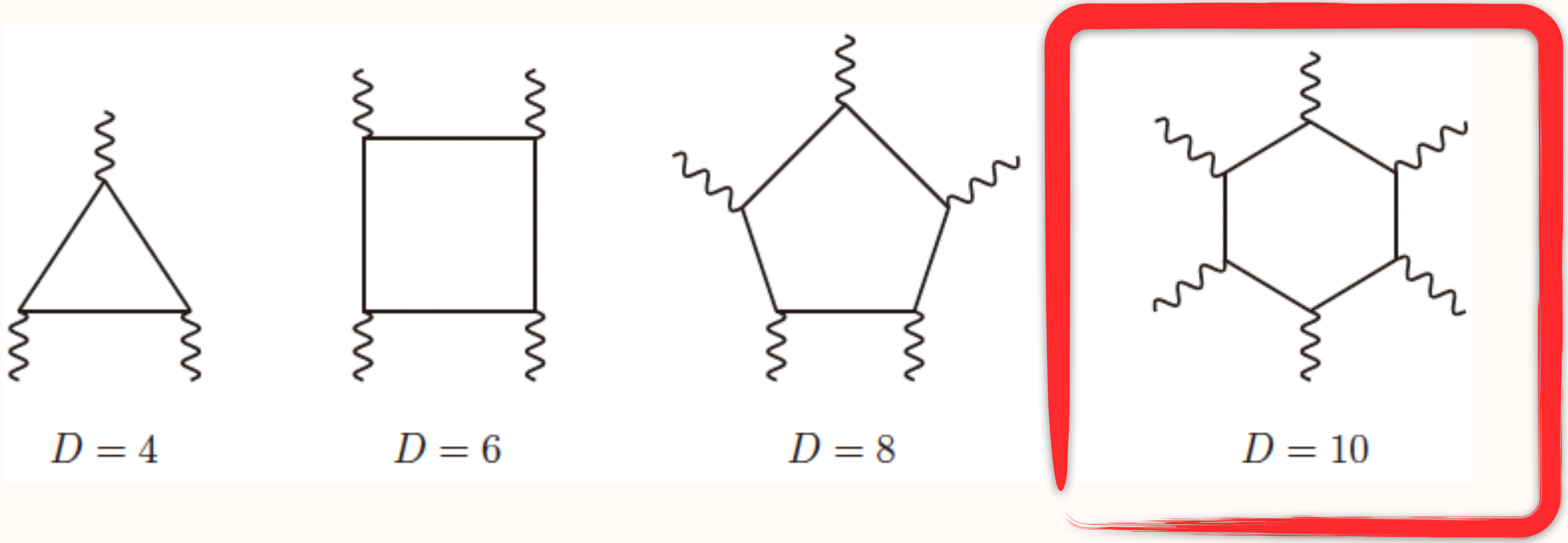

In 10D, the hexagon anomaly. It is cancelled by the previous GS term.

Green-Schwarz mechanism:

The gravity anomaly in 10D requires 496 spin-1/2 fields. Possible non-Abelian gauge groups are rank 16 groups SO(32) and E8xE8'. The anti-symmetric field B_{MN} has field strength (in diff notation), $H = dB + w_{3Y}{}^0 - w_{3L}{}^0$:SO(32). Three indices matched.

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$$-\frac{3\kappa^2}{2g^4\,\varphi^2}H_{MNP}H^{MNP}, \text{ with } M,N,P=\{1,2,\cdots,10\}$$

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The dual of H is the so-called MI-axion [Witten (1984)]

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$$S_1' \propto -\frac{c}{10800}\left\{H_{\mu\nu\rho}A_\sigma\,\epsilon^{\mu\nu\rho\sigma}\epsilon^{ijklmn}\langle F_{ij}\rangle\langle F_{kl}\rangle\langle F_{mn}\rangle+\cdots\right\}\to\frac{1}{3!}\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho}A^\sigma$$

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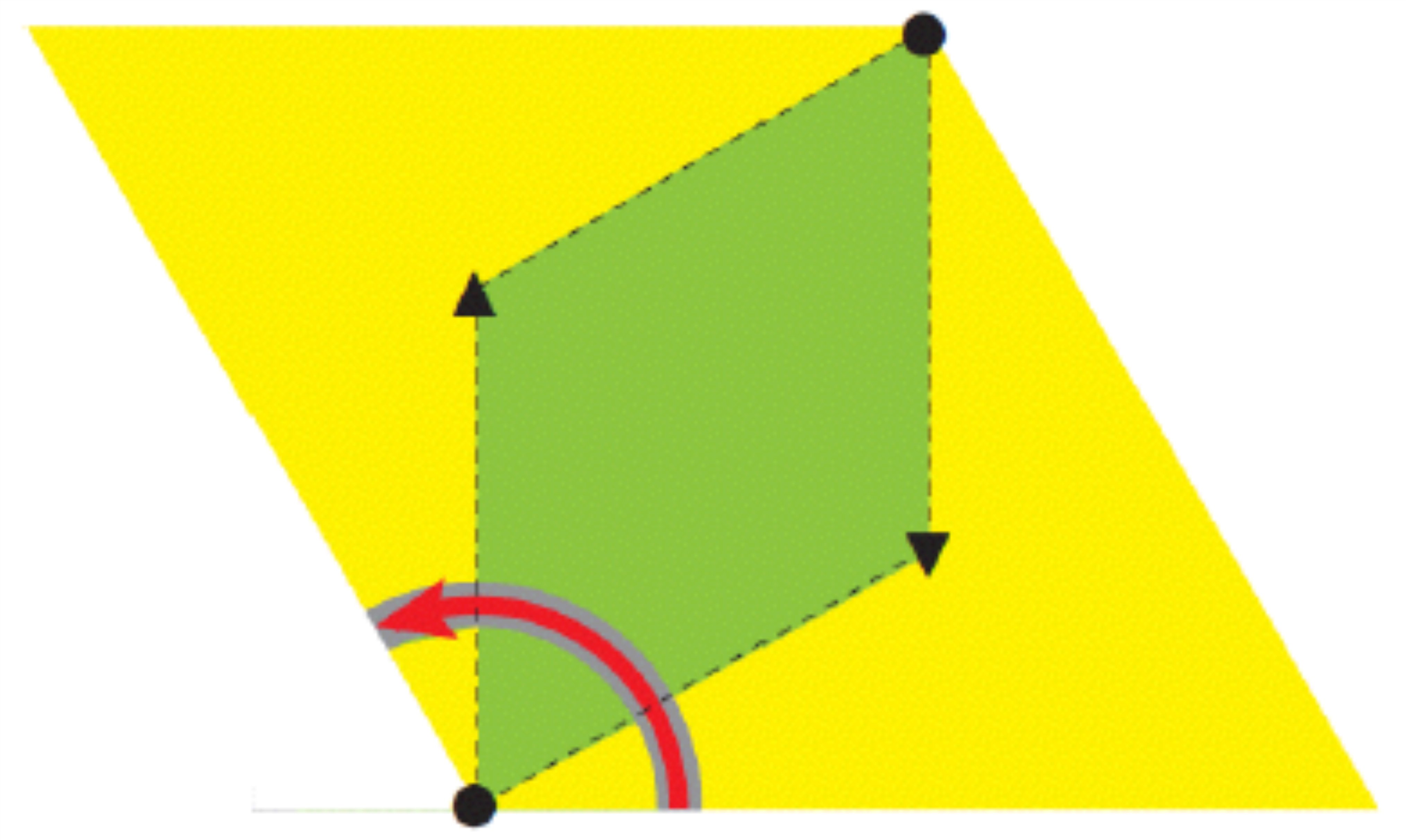

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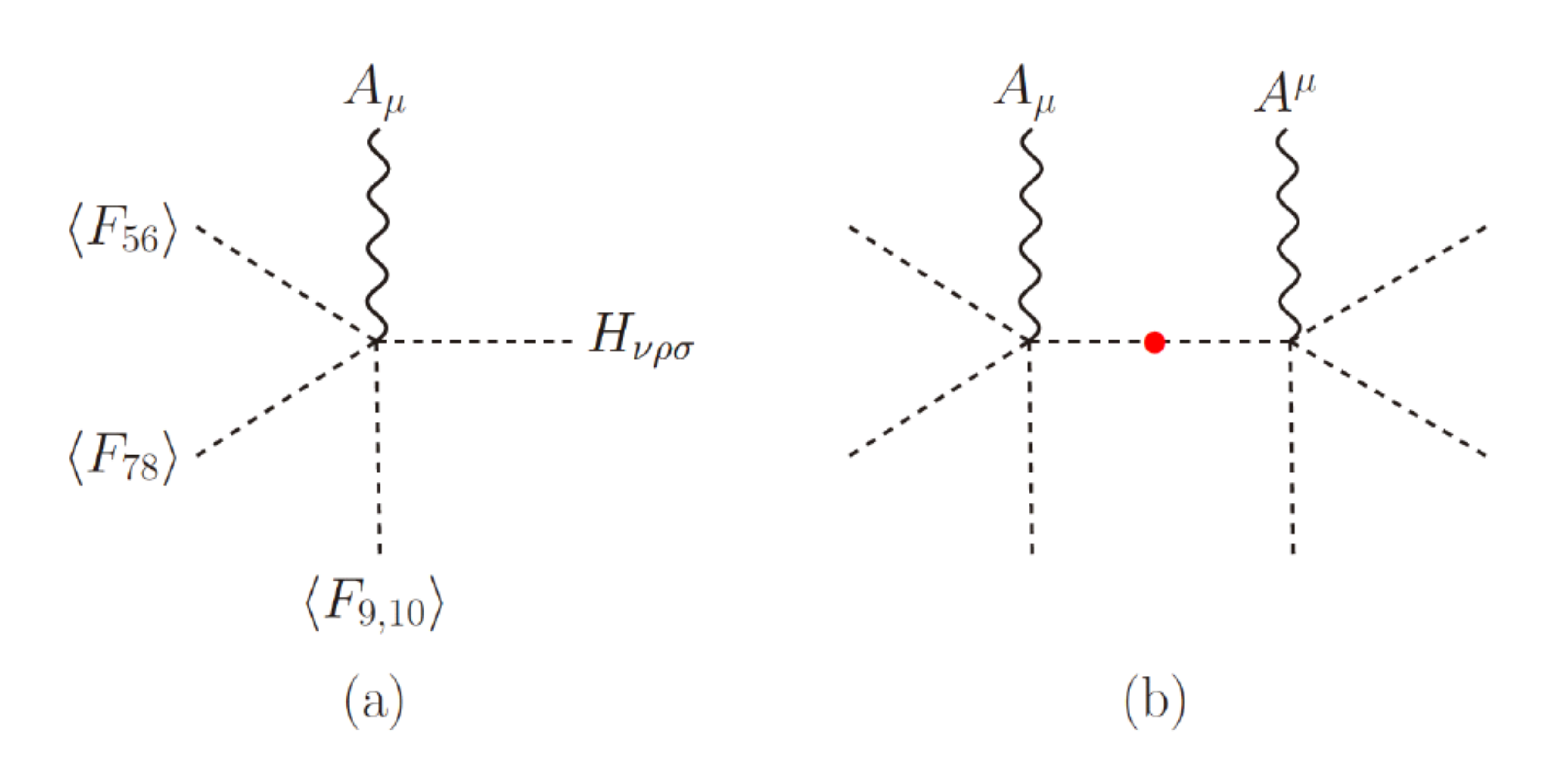
$$\frac{1}{2\cdot 3! M_{MI}^2}H_{\mu\nu\rho}H^{\mu\nu\rho},\ \text{with}\ \mu,\nu,\rho=\{1,2,3,4\}.$$



$$M_{MI}A_\mu\partial^\mu a_{MI}$$

$$\frac{1}{2}M_{MI}^2A_\mu A^\mu$$

$$\frac{1}{2}M_{MI}^2(A_\mu+\frac{1}{M_{MI}}\partial_\mu a_{MI})^2$$

One may look this in the following way.

The10 supergravity quantum field theory with SO(32) and E8xE8' gauge groups has gauge and gravity anomalies. Let us believe that string theory is consistent, effectively removing all divergences, i.e. removing all anomalies. The point particle limit of 10D string theory should not allow any anomalies. There must be some term in the string theory removing all these anomalies. It is the GS term. In strong int., breaking chiral symmetry, viz. the Wess-Zumino term removing anomalies by some term involving pseudoscalar fields.

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Thus, since the MI axion is a real spin-0 particle, f_a can be related to the string scale.

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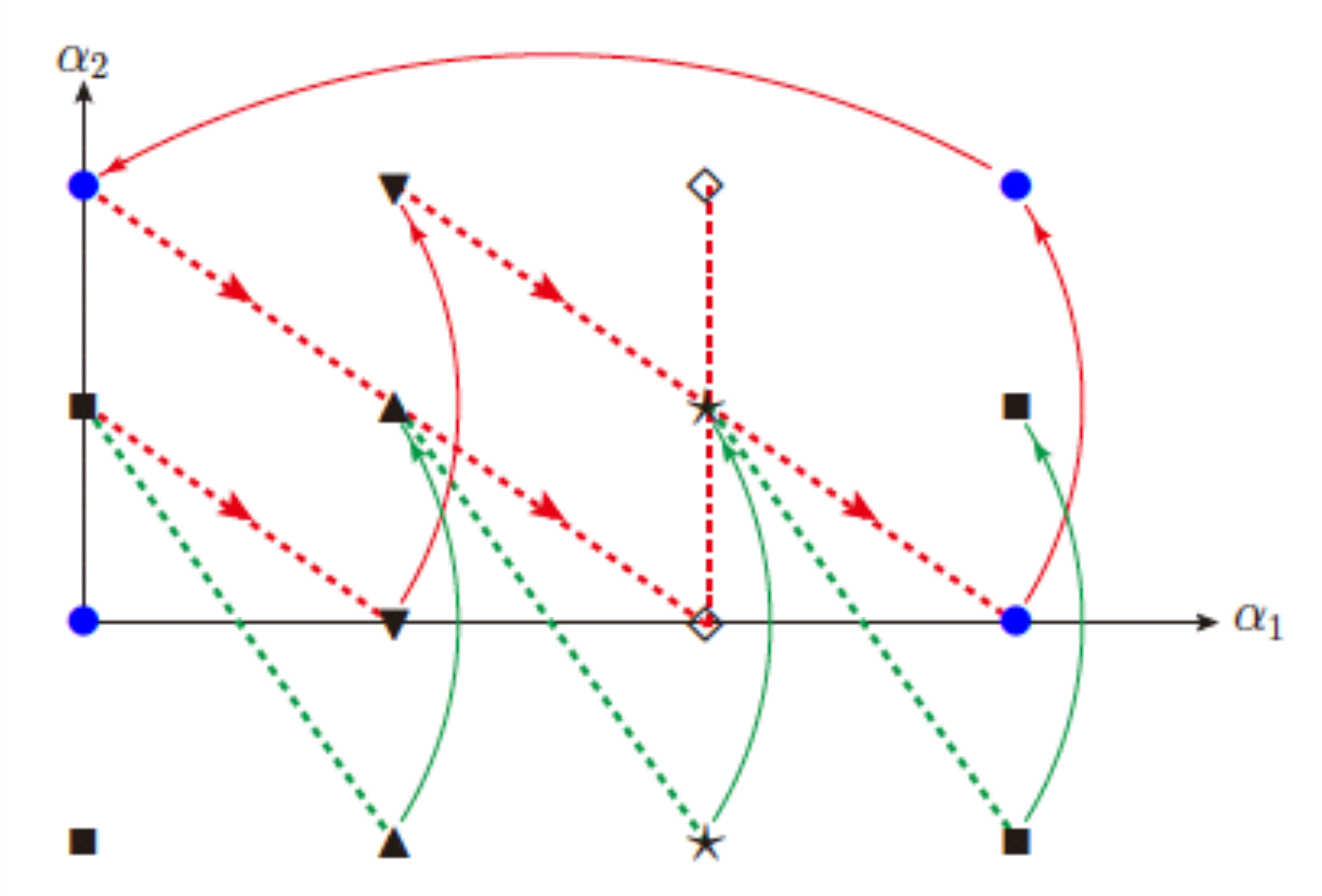

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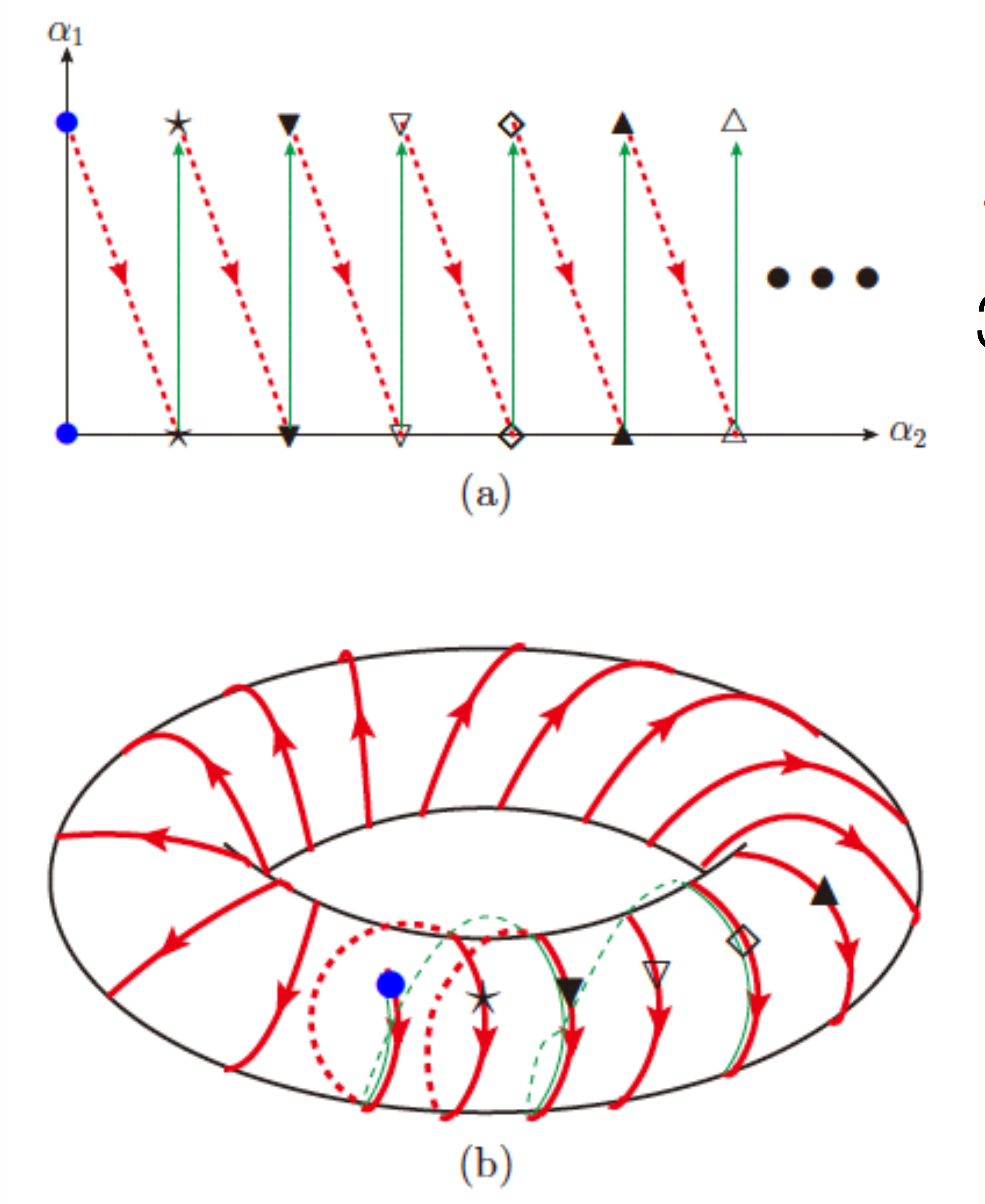

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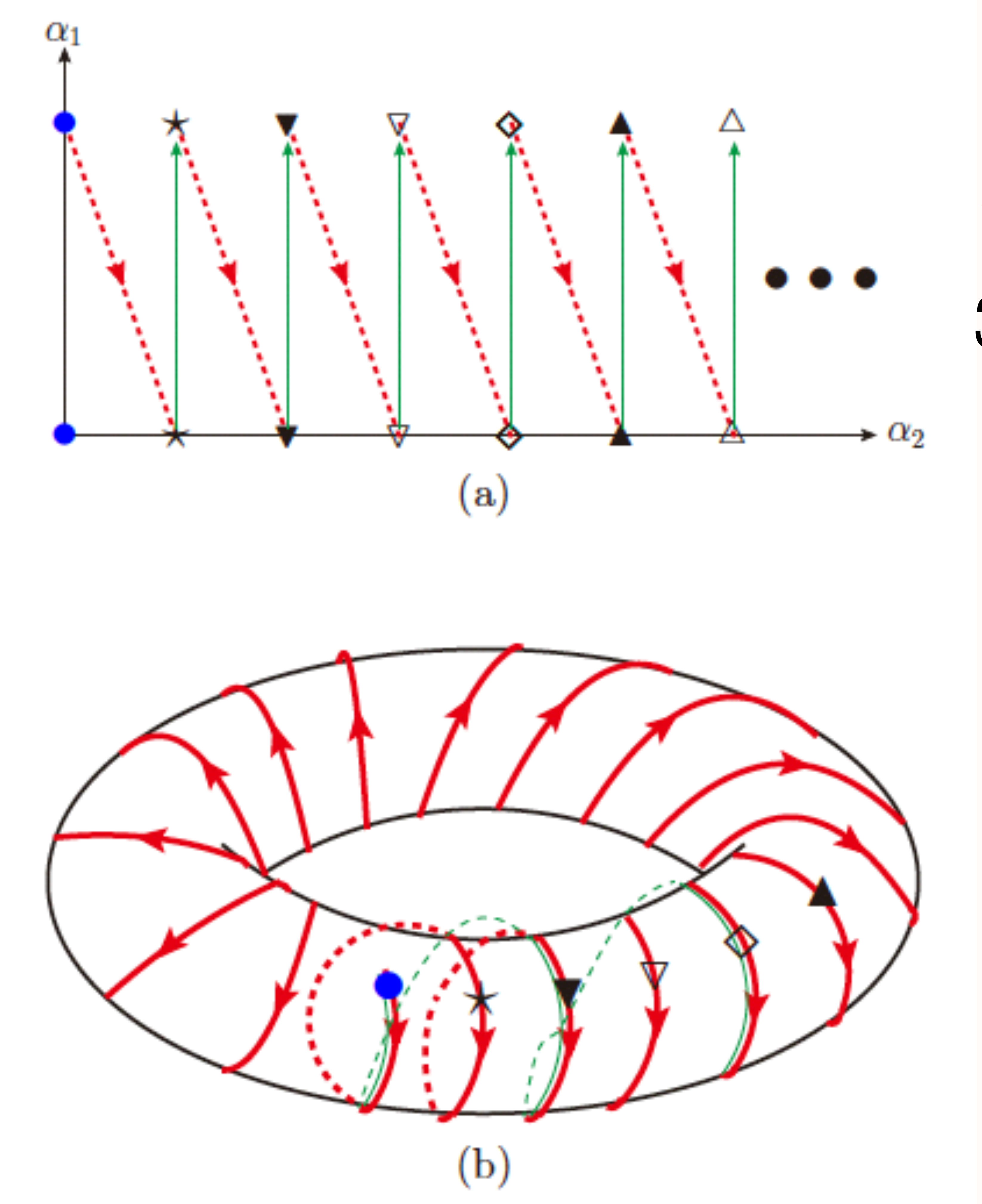

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4. In the example of 1710.08454, based on the model of Huh-Kim-Kyae, sum of U(1)-$SU(3)_c^2$ anomaly is $3492=2^2 x 3^2 x 97$. So, there is a great chance that M_{MI} : $f_{\text{phi at st scale}}$ of 3491: 3493, 3497, 3499 etc will lead to N_{DW}=1 because they are relatively prime. Thus, the global symmetry is determined purely from the VEVs at the string scale.

Thus “invisible” axion from anomalous U(1) satisfies the requirements for the intermediate f_a.

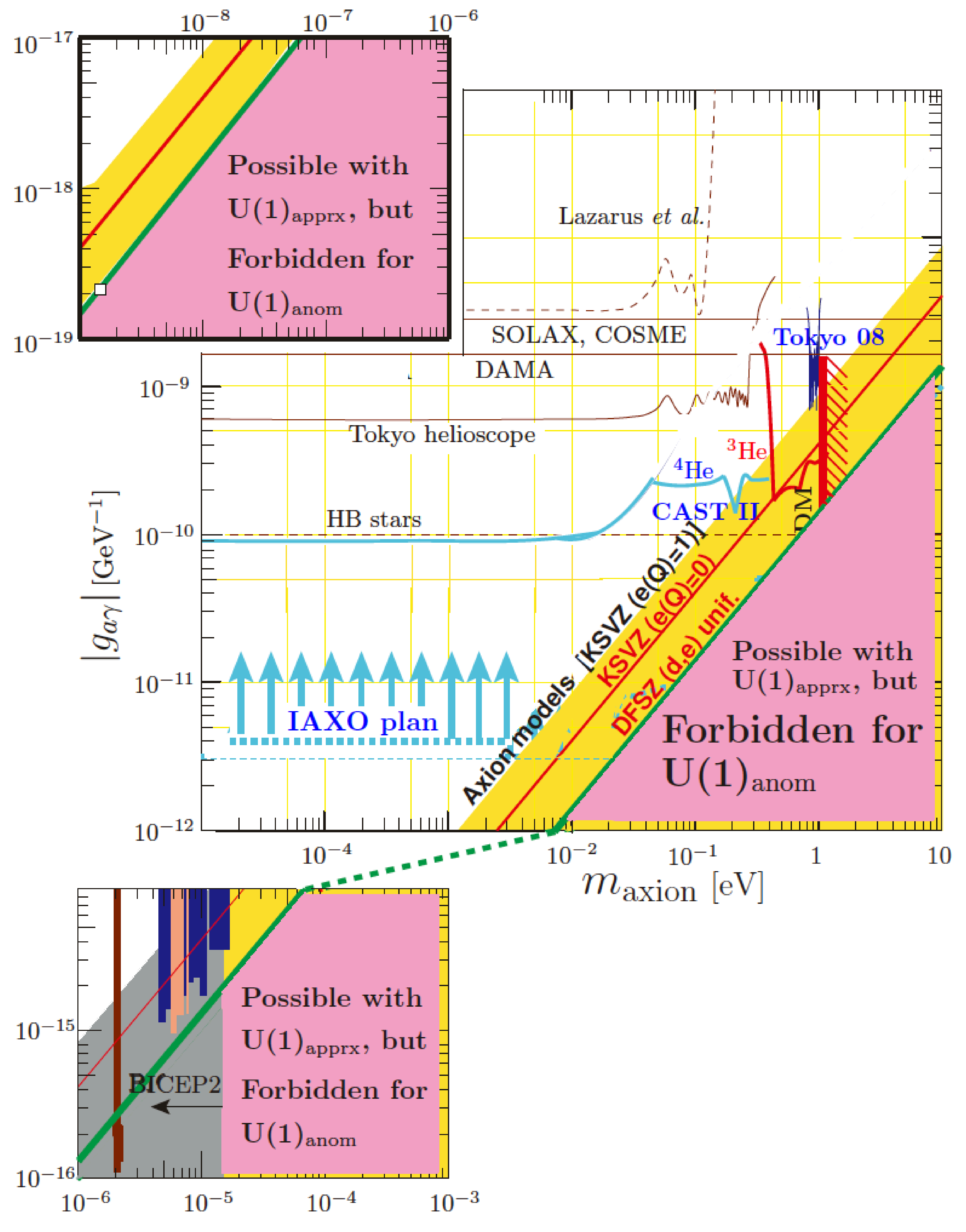
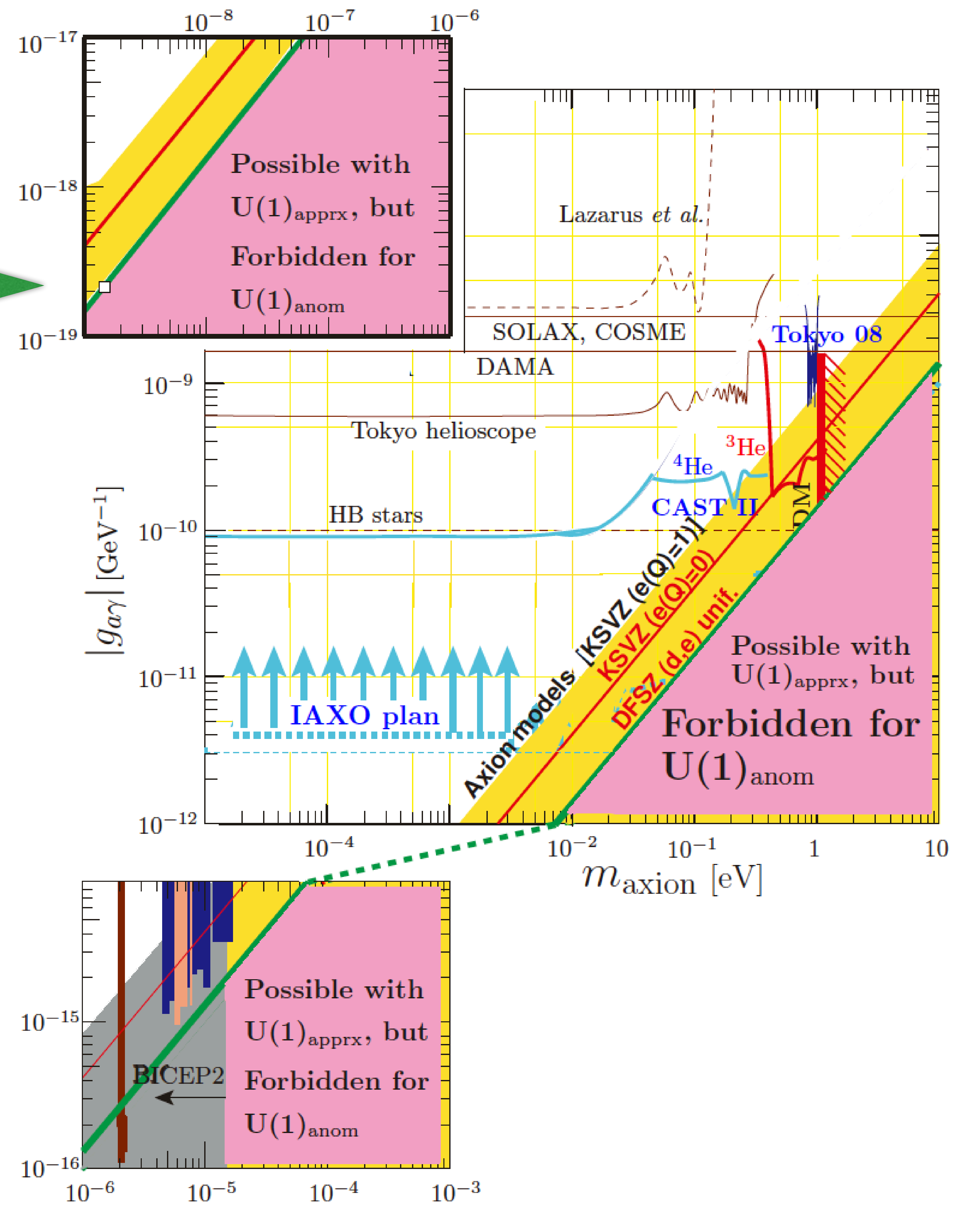


Fig. 8. The $g_{a\gamma}(= 1.57 \times 10^{-10}\, c_{a\gamma\gamma})$ vs. m_a plot.[80,81]

Choi-Kim, 1985

f_{MI}



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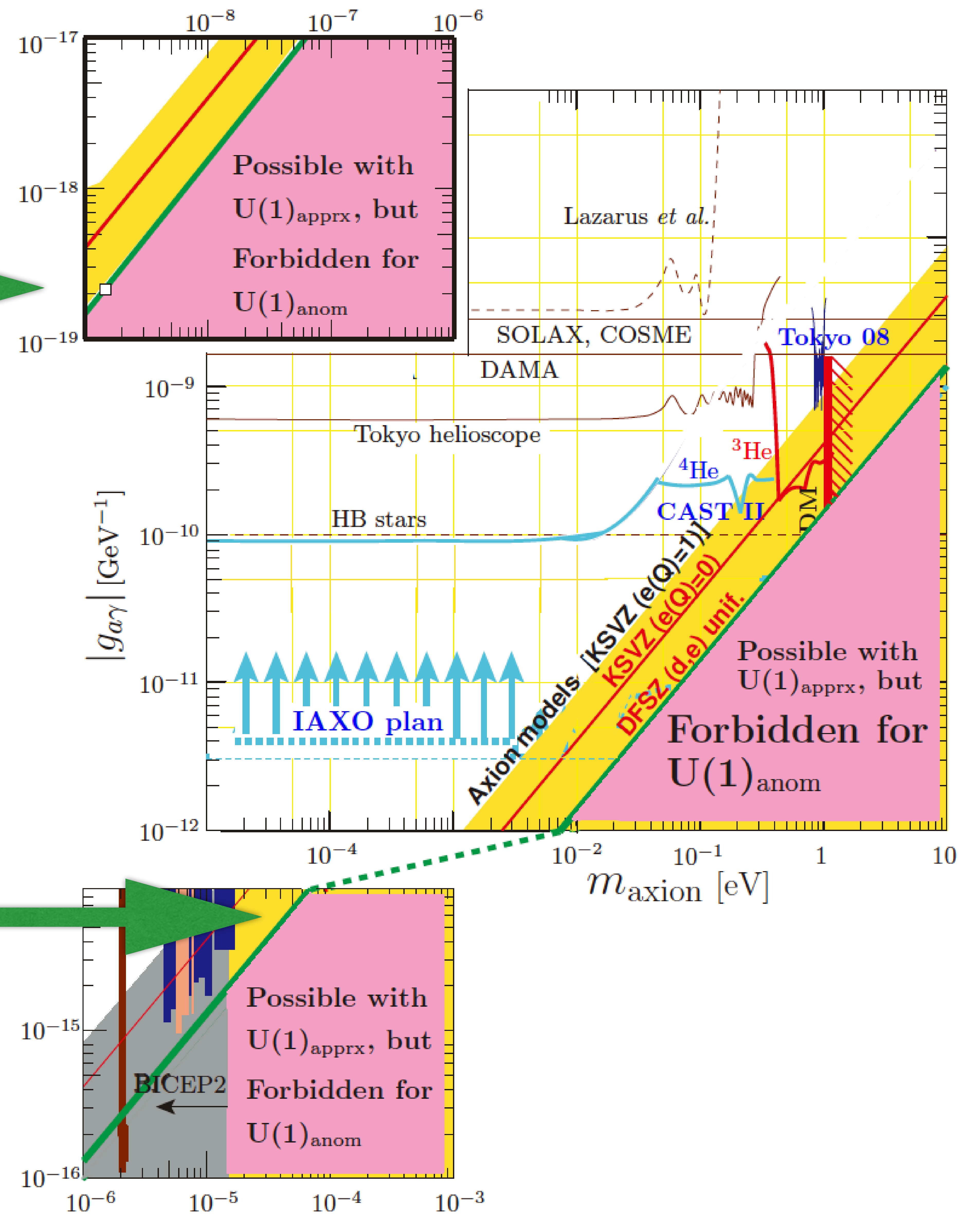
f_{MI}

But, we need “invisible” axion here

f_a



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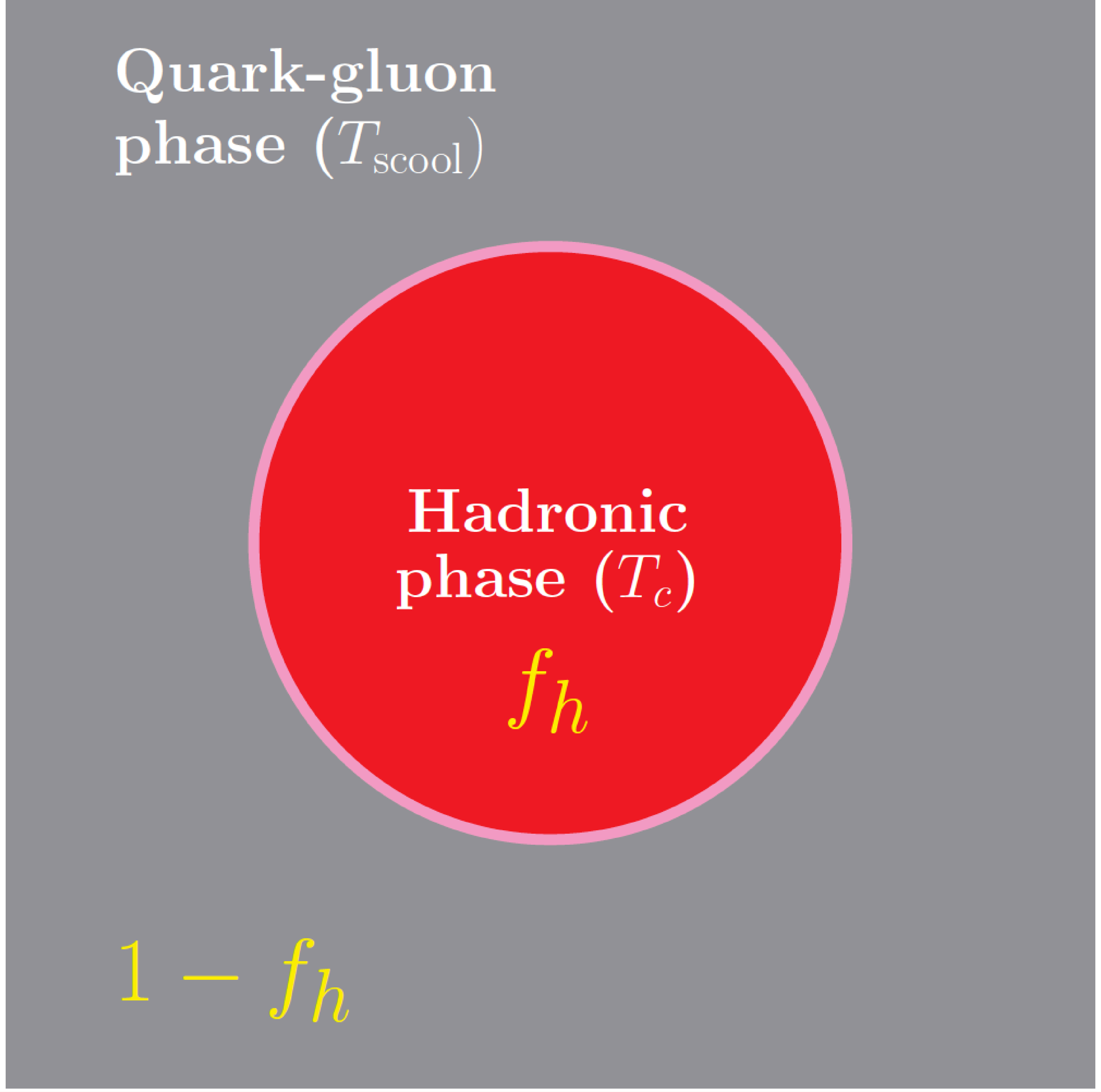
Quark-gluon
phase ($T_{\rm scool}$)
Hadronic
phase (T_c)
f_h
$1 - f_h$

$$\text{Before}\begin{cases}\rho = \frac{\pi^2}{30}\, g_*^i T^4 \\ s = \frac{2\pi^2}{45}\, g_*^i T^3\end{cases}$$

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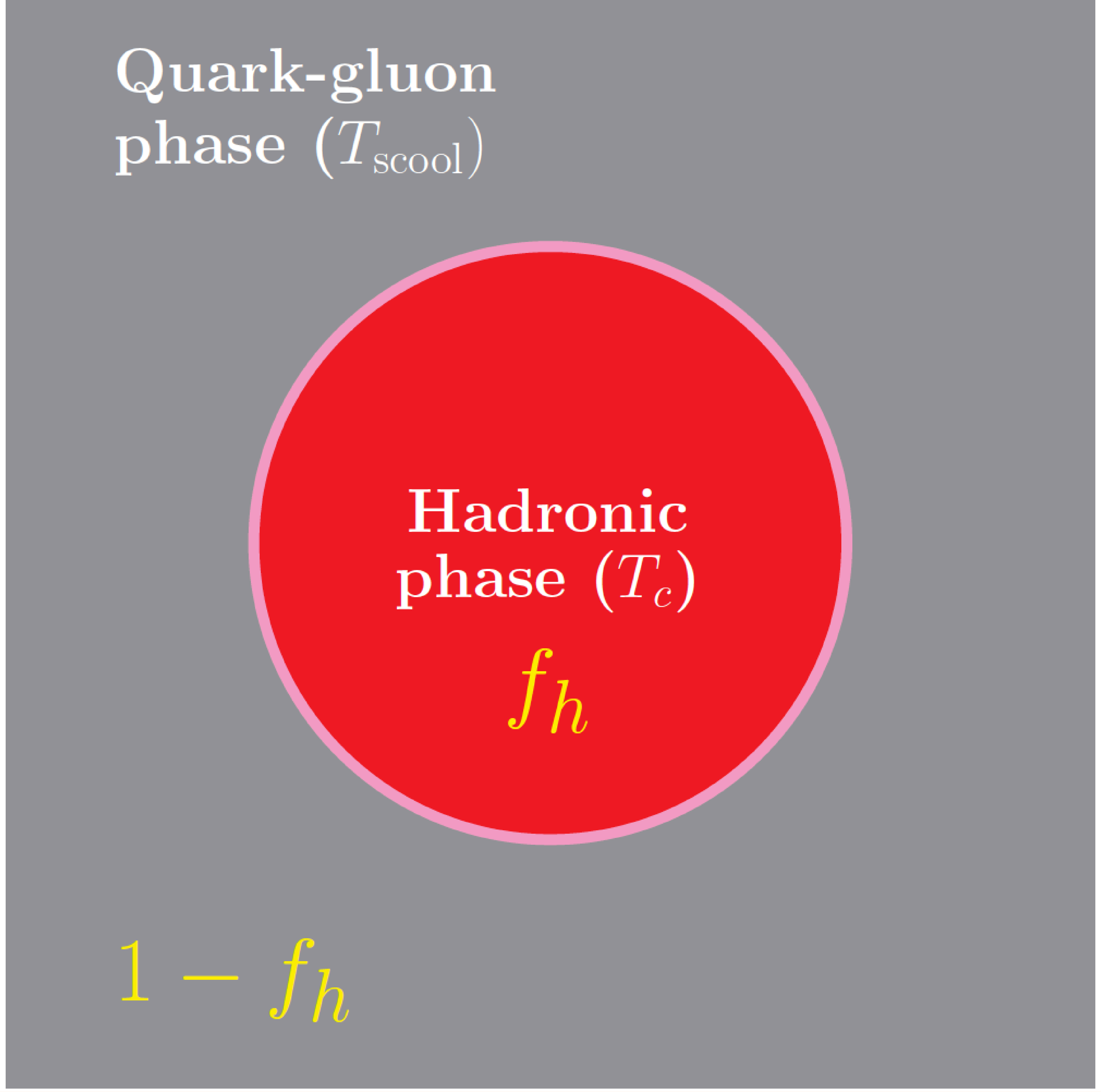
$$g_*^i = 51.25, \;\; g_*^f = 17.25.$$



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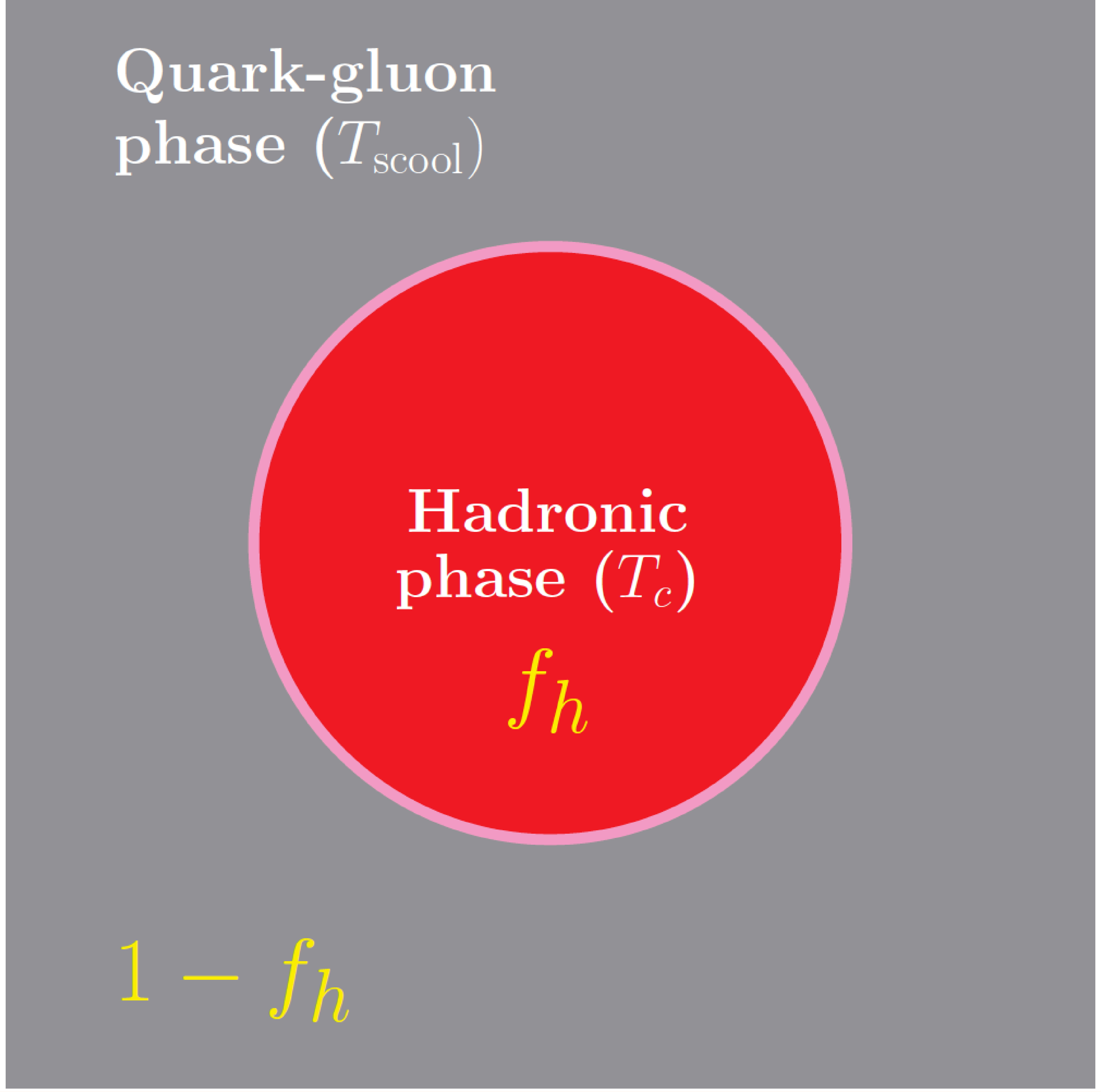

$$(\Delta S)_{\rm light} = (g_*^i - g_*^f)\frac{\pi^2}{30}T^3 - (g_*^i - g_*^f)\frac{2\pi^2}{45}T^3 = -\frac{\pi^2}{90}(g_*^i - g_*^f)T^3$$

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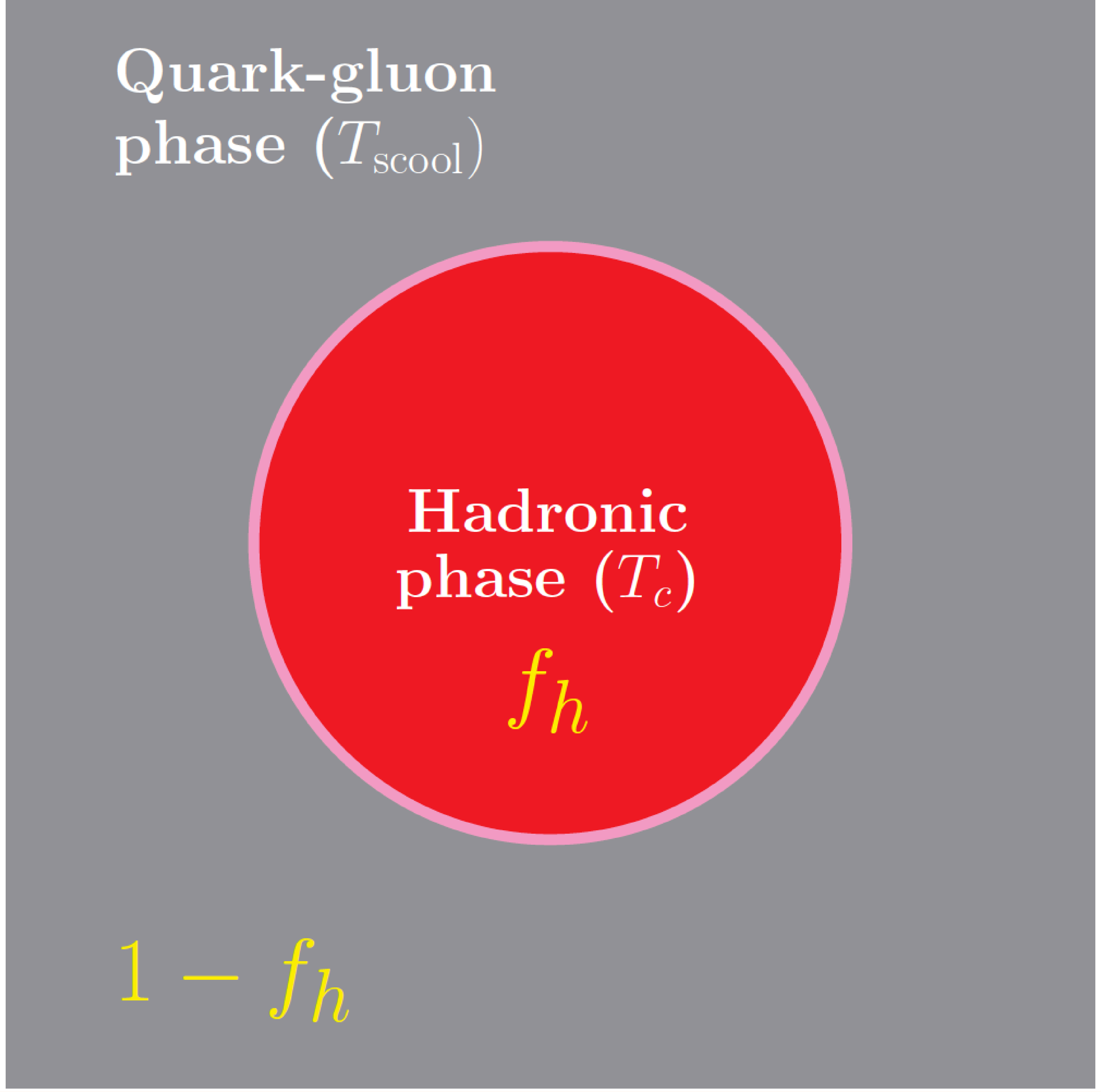

$$(\Delta S)_{\rm light} = (g_*^i - g_*^f)\frac{\pi^2}{30}T^3 - (g_*^i - g_*^f)\frac{2\pi^2}{45}T^3 = -\frac{\pi^2}{90}(g_*^i - g_*^f)T^3$$

$$\frac{df_h}{dt} = \alpha(1 - f_h) + \frac{3}{(1 + C f_h(1 - f_h))(t + R_i)} f_h$$

$$\Delta S = -\frac{\pi^2}{90}(g_*^i - g_*^f)T^{-3} + g_*^f \frac{2\pi^2}{45}(T_c^3 - T^{-3})$$
$$= -\frac{\pi^2}{90}(g_*^i + 3g_*^f)T^{-3} + g_*^f \frac{2\pi^2}{45}(T_c^3) \geq 0.$$

For the energy condition, we require that the h-phase bubble does not to shrink

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by supercooling

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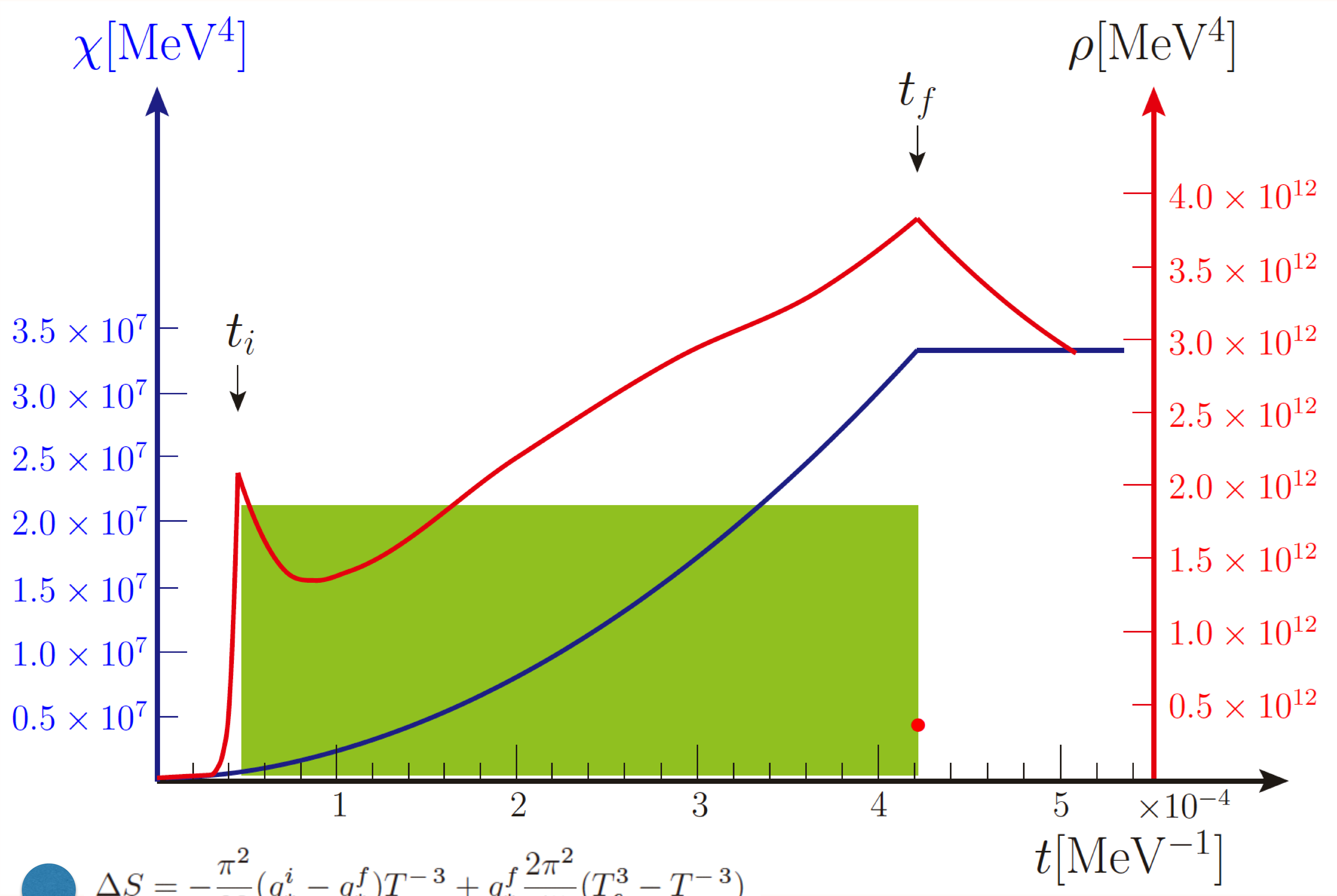

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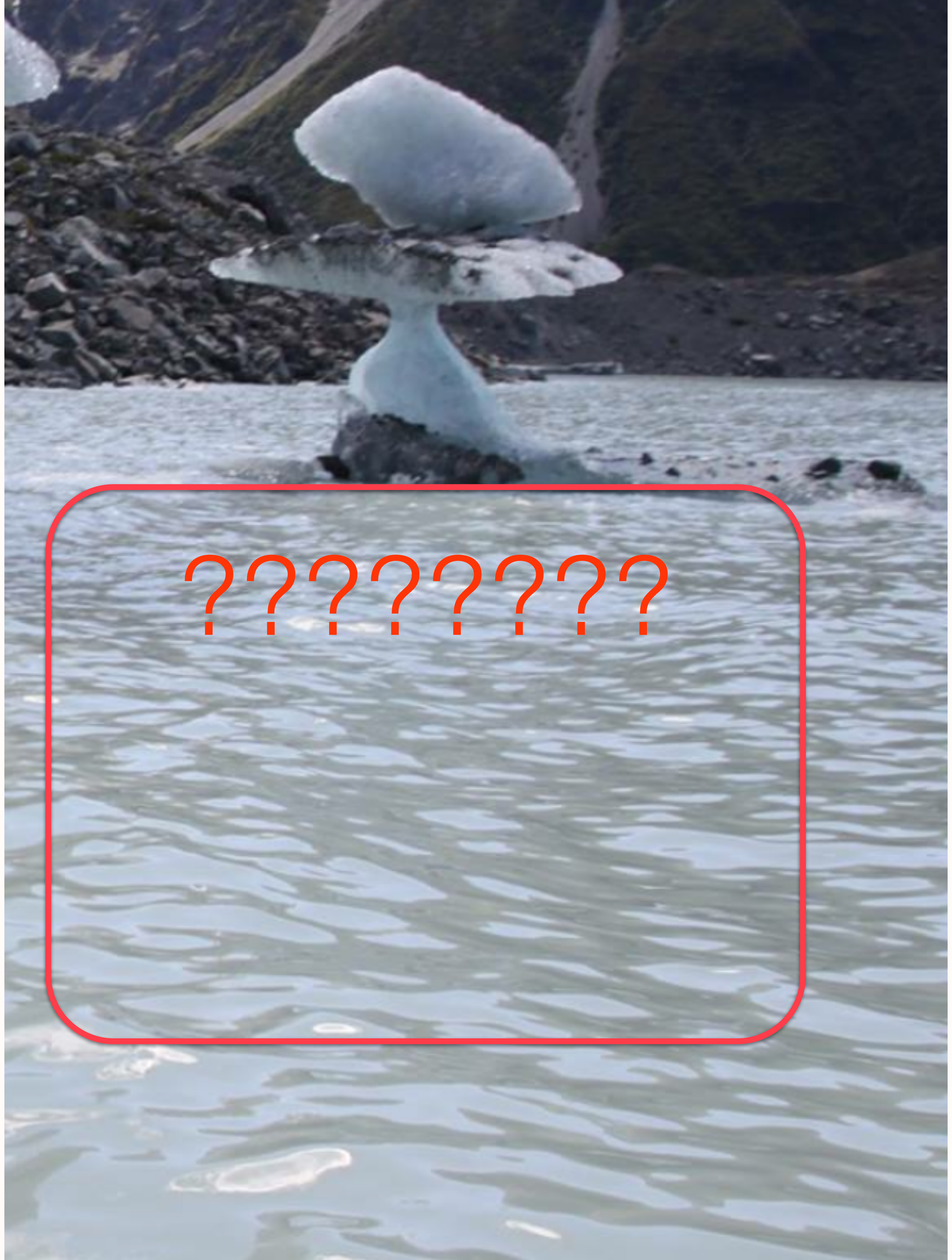
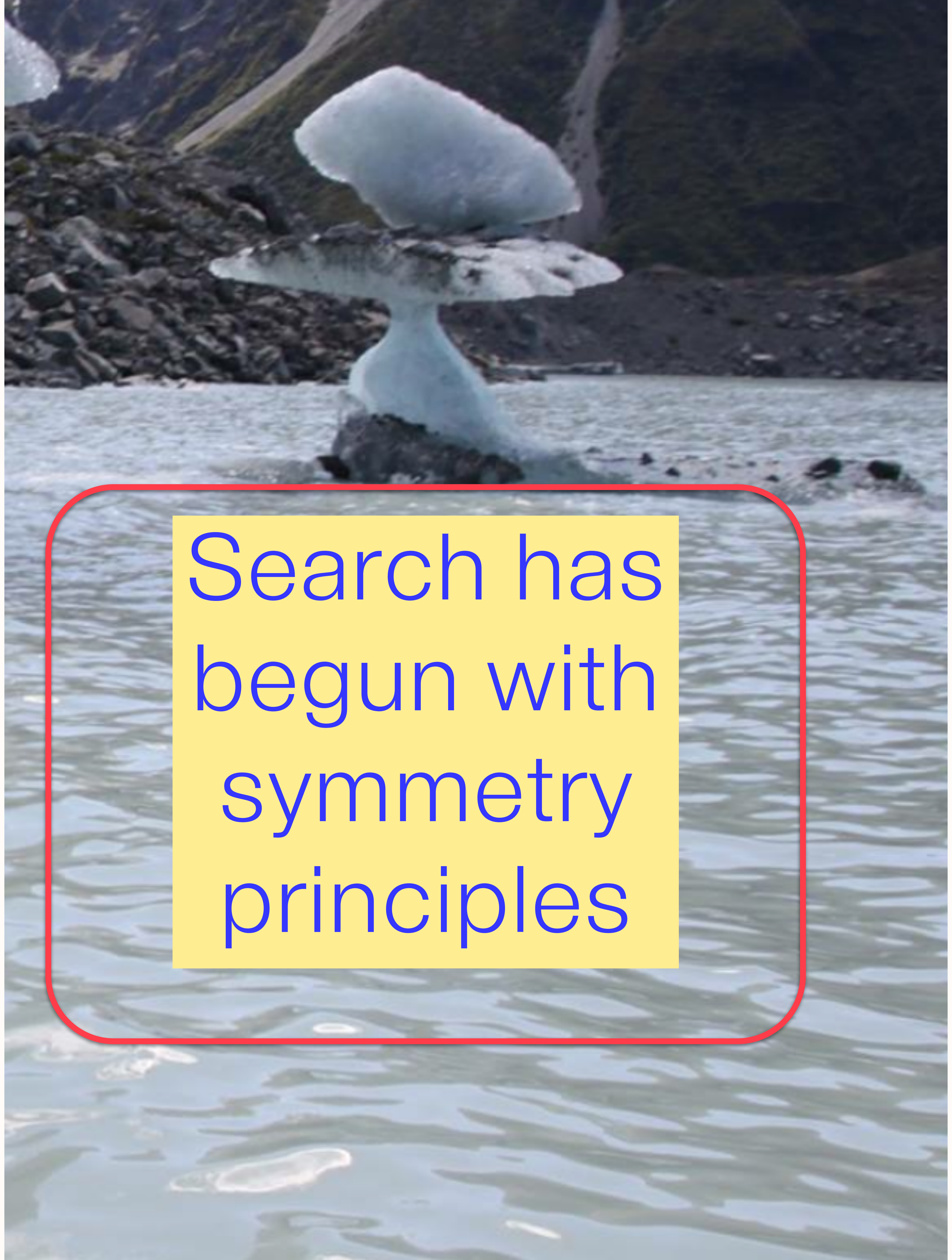
6. Approximate global symmetry




Search has begun with symmetry principles

Symmetry is beautiful: a framework, beginning with Gross' grand design.

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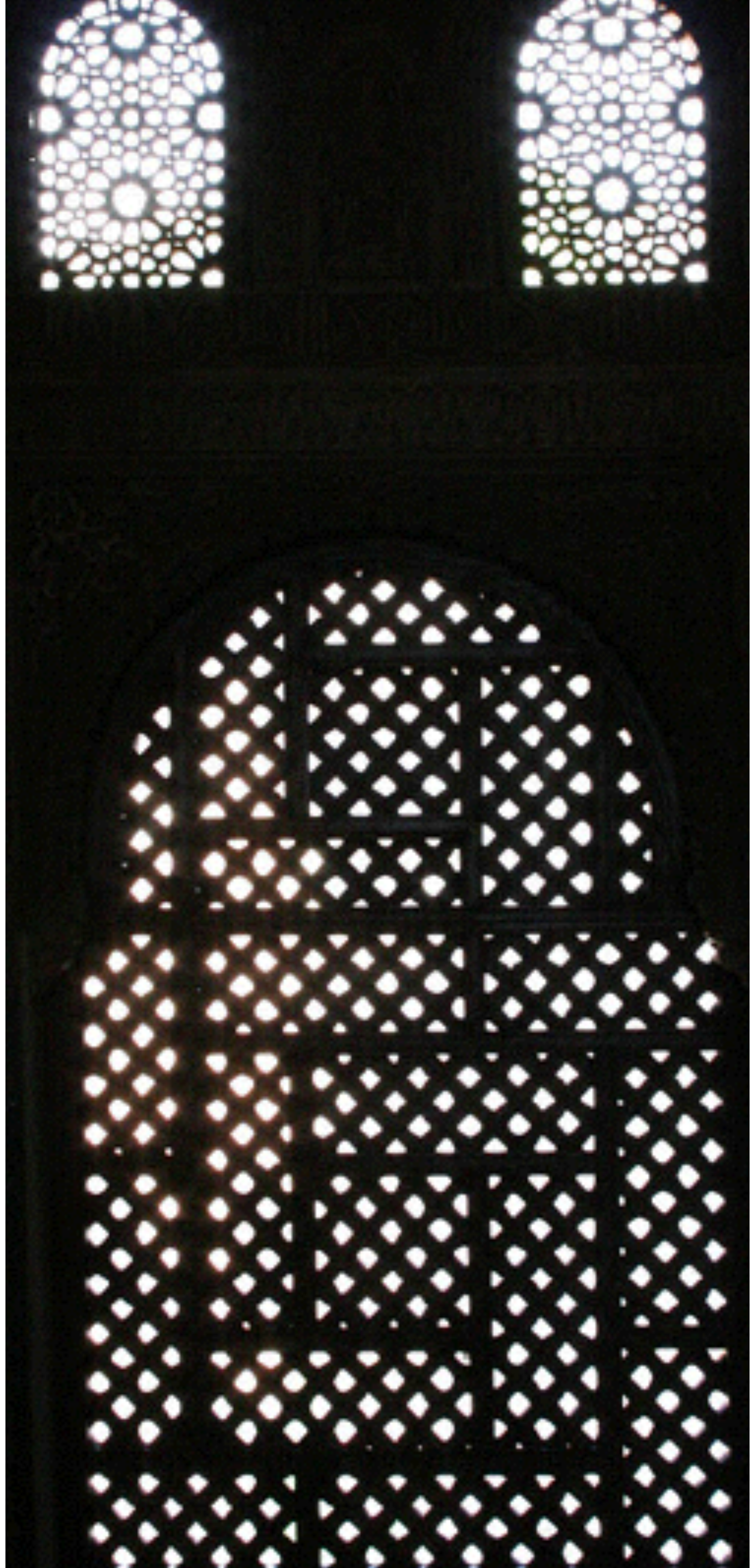
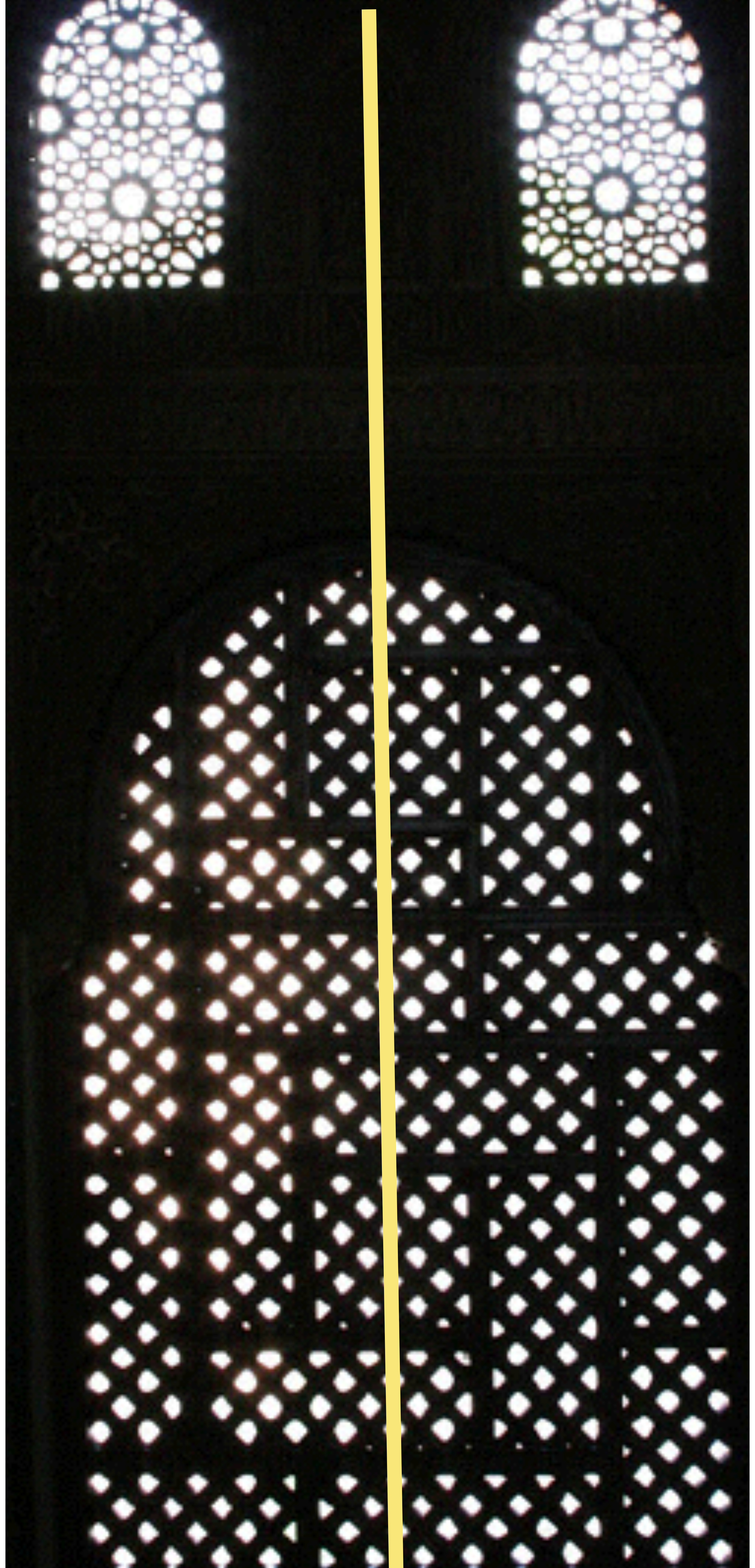


Symmetry is beautiful: a framework, beginning with Gross' grand design.

Parity:



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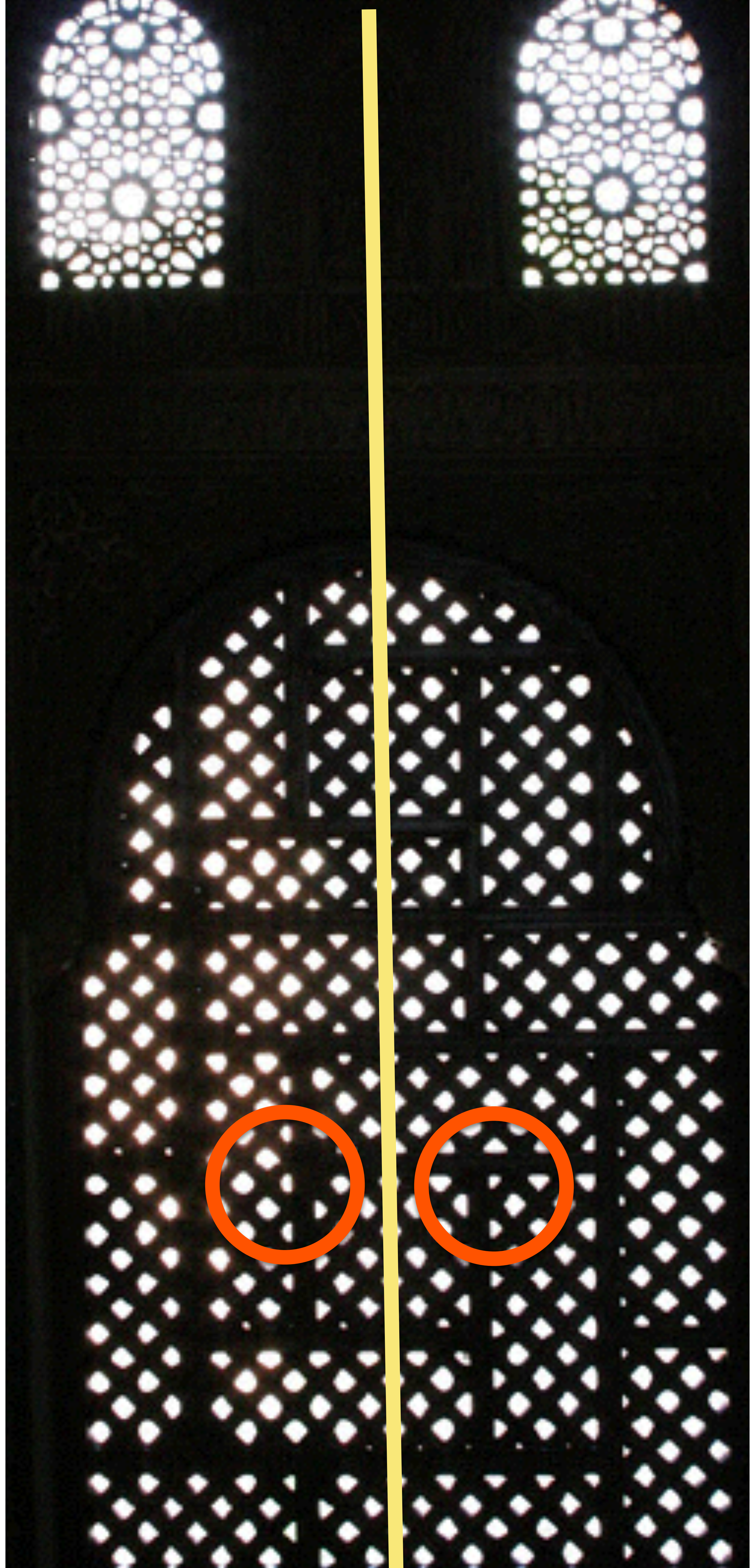
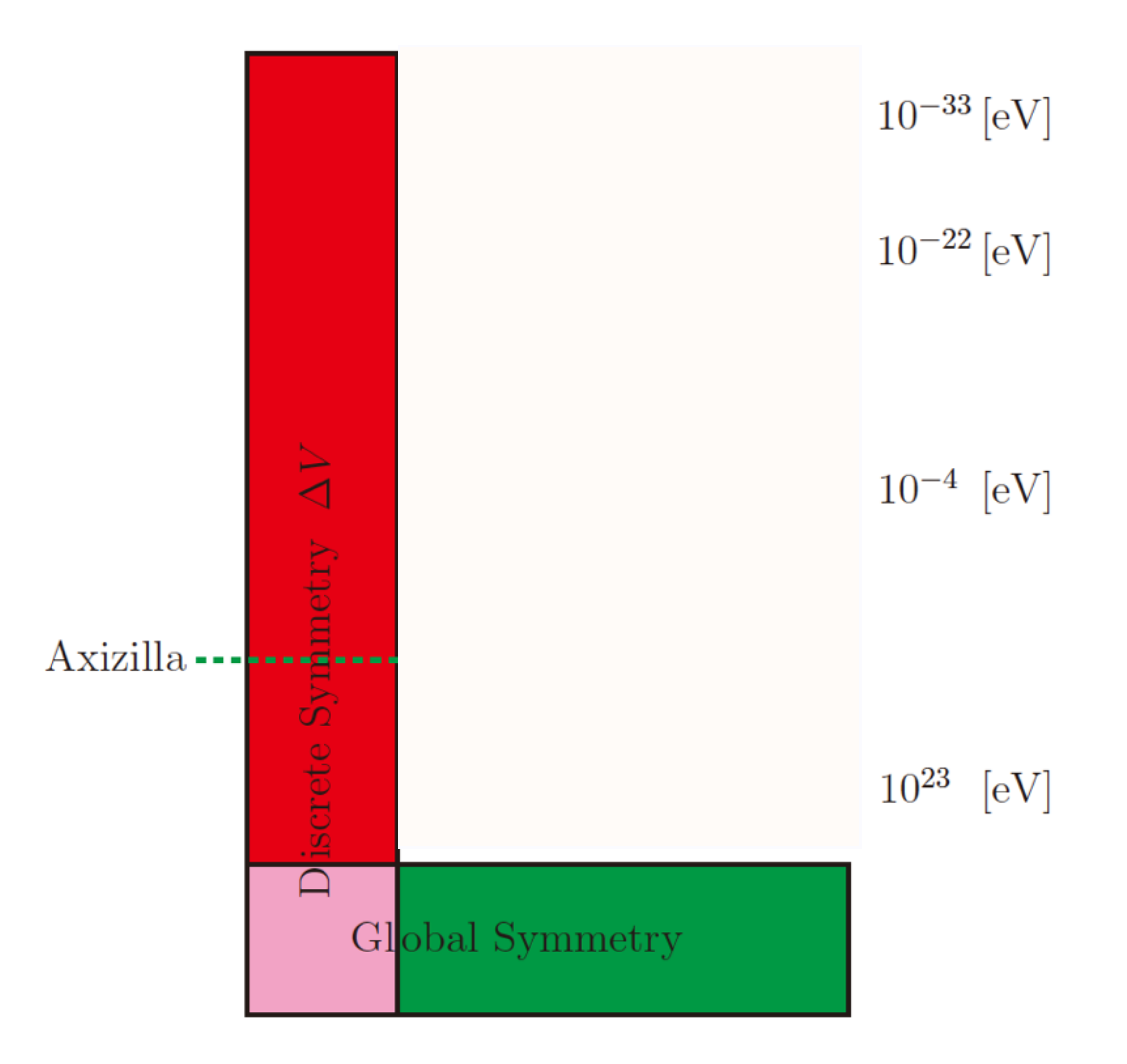
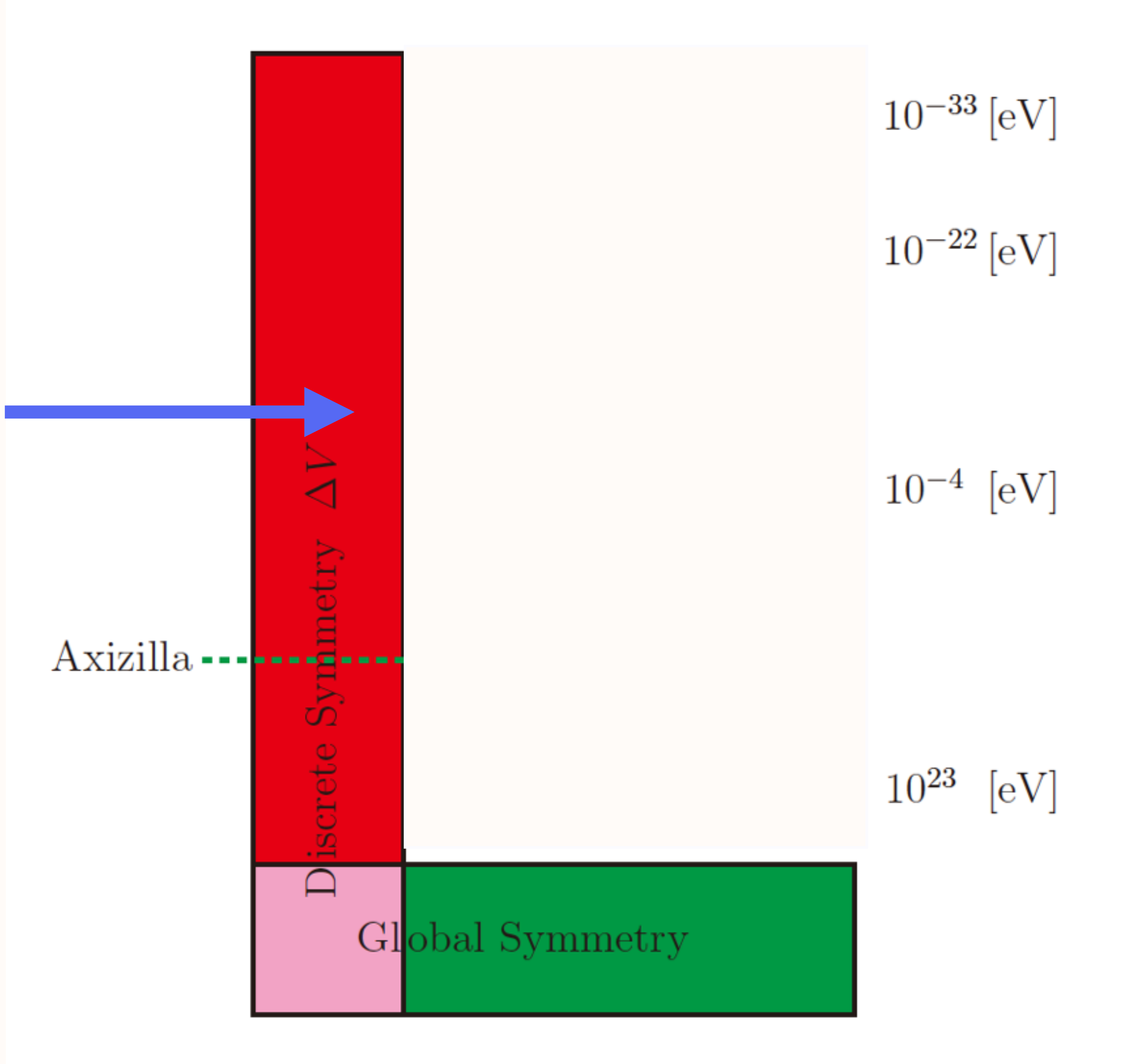


Parity: Slightly broken!



10^{-33} [eV]
10^{-22} [eV]
10^{-4} [eV]
10^{23} [eV]
Discrete Symmetry ΔV
Axizilla
Global Symmetry

Except the anomalous U(1), any global symmetry does not have anomalies from string theory. So, this V is present.



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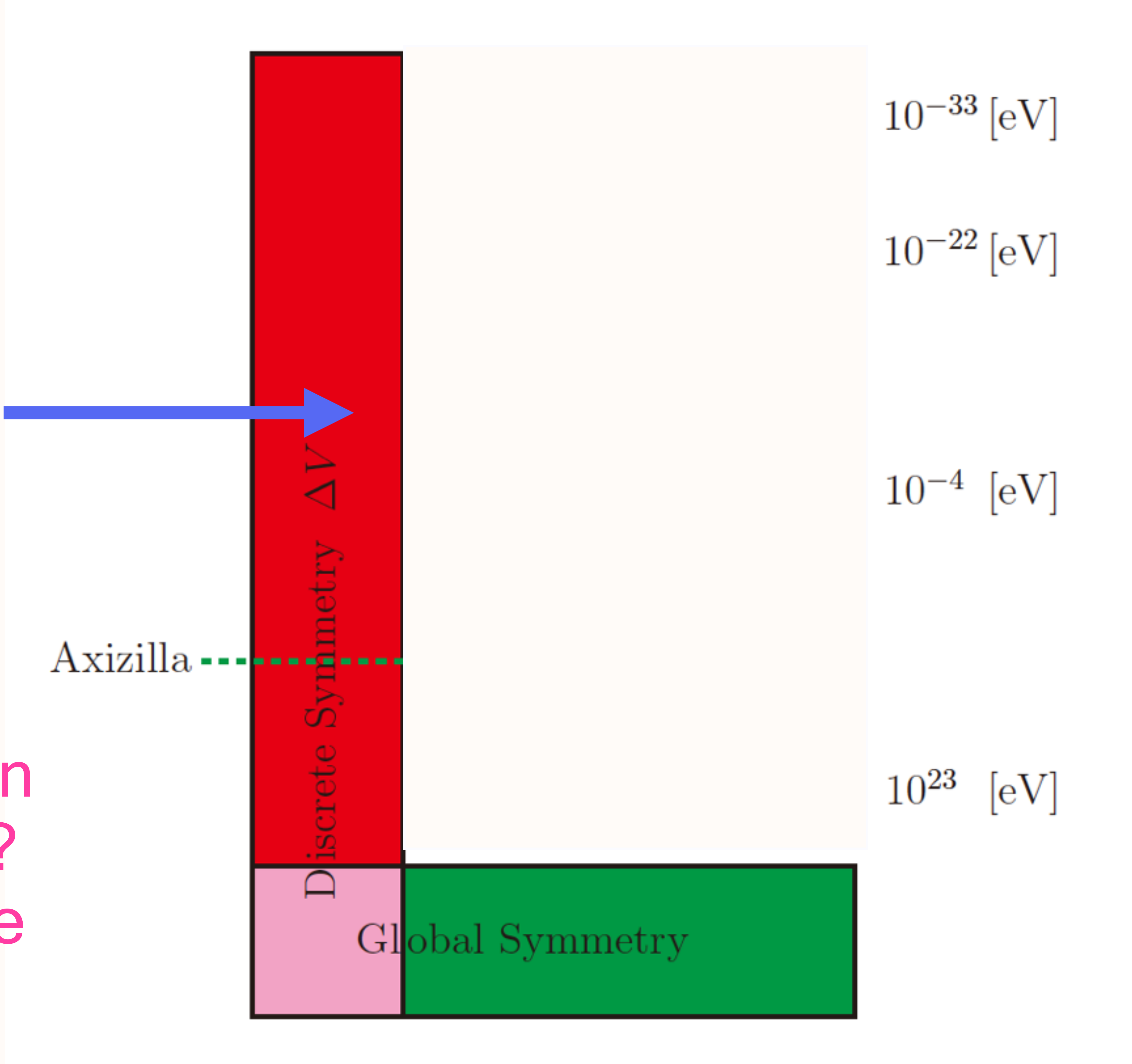

Still the question is at what level? If one allows the discrete symmetry from string.

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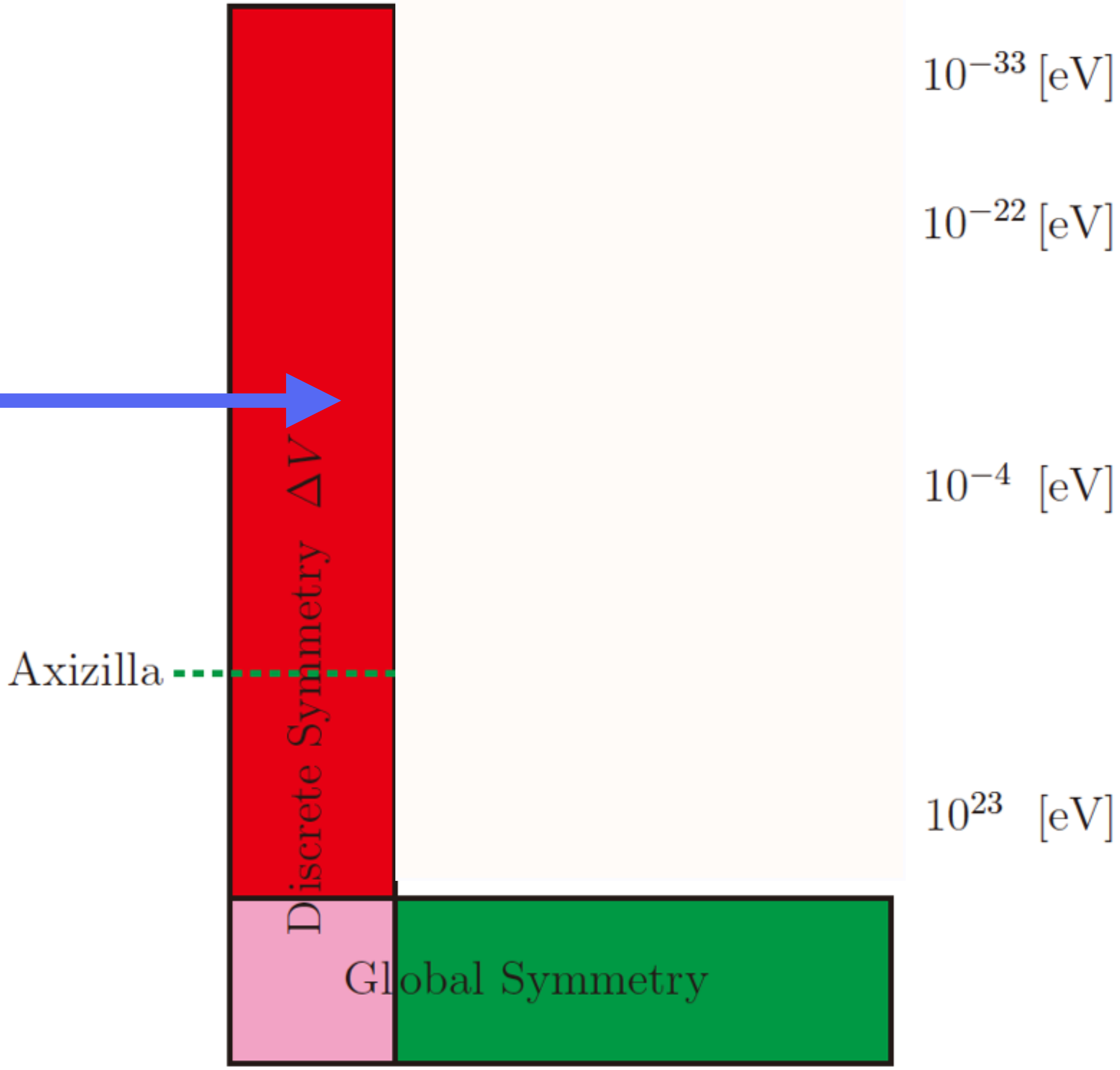

Quintessential axion

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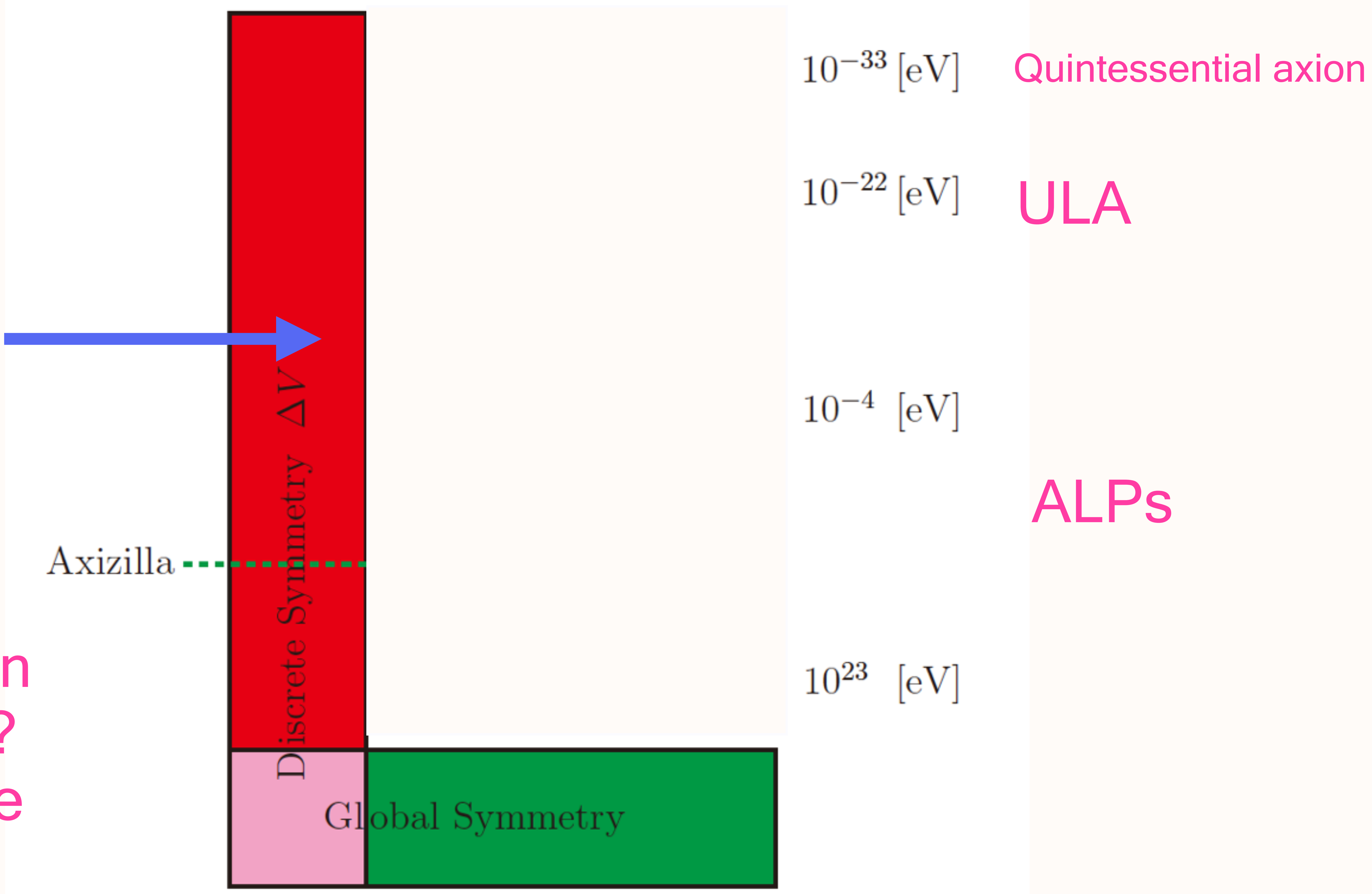

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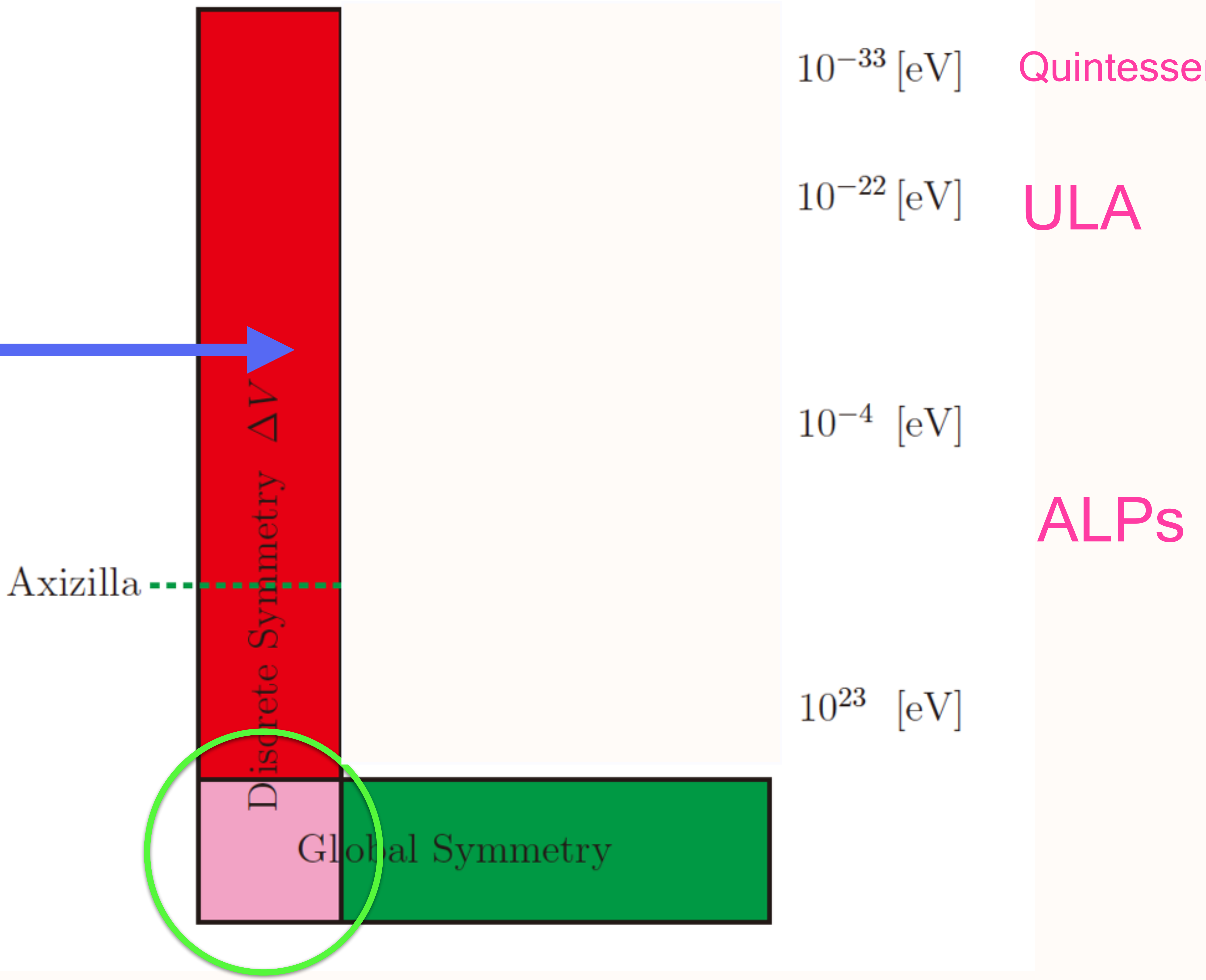

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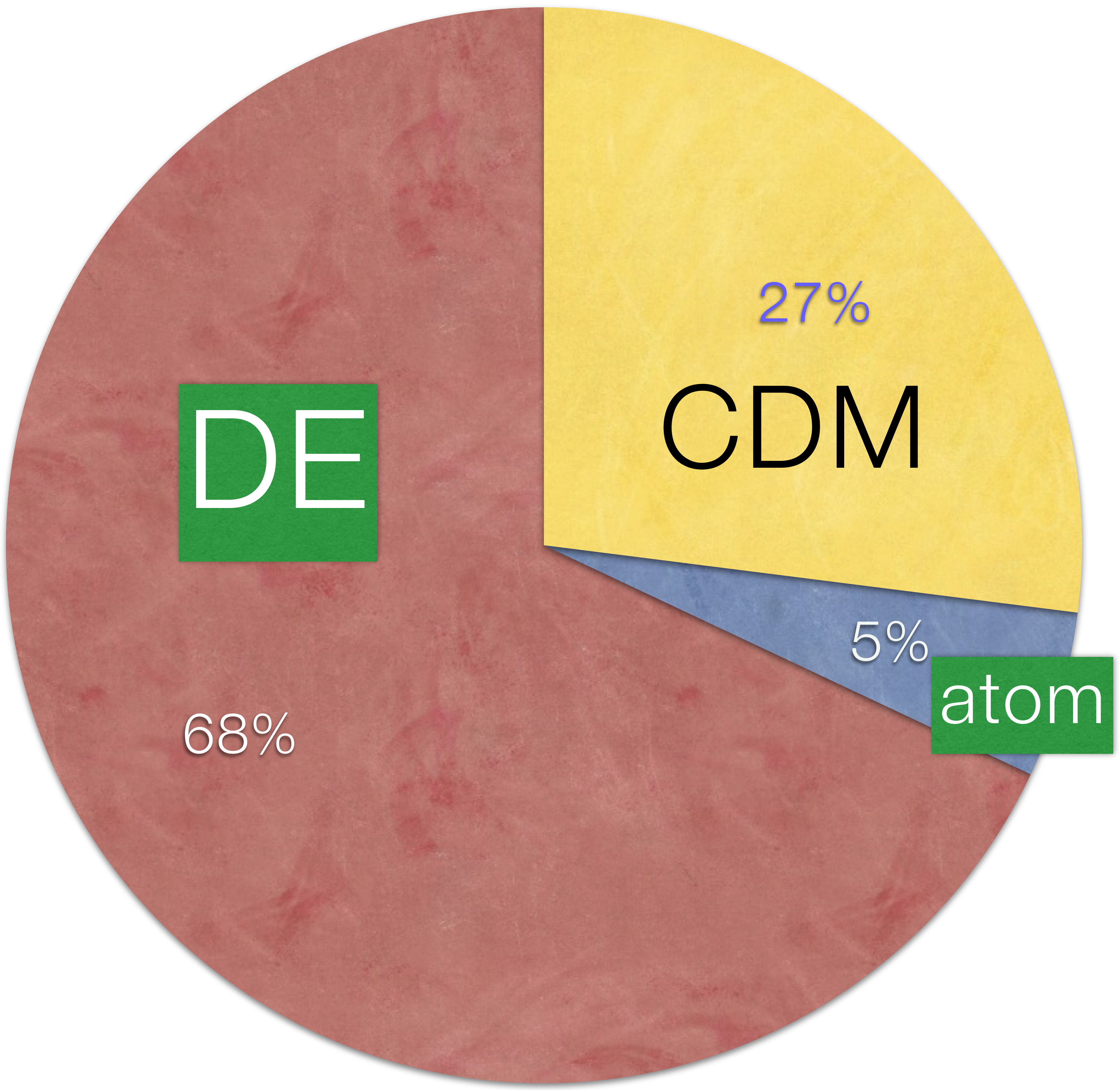
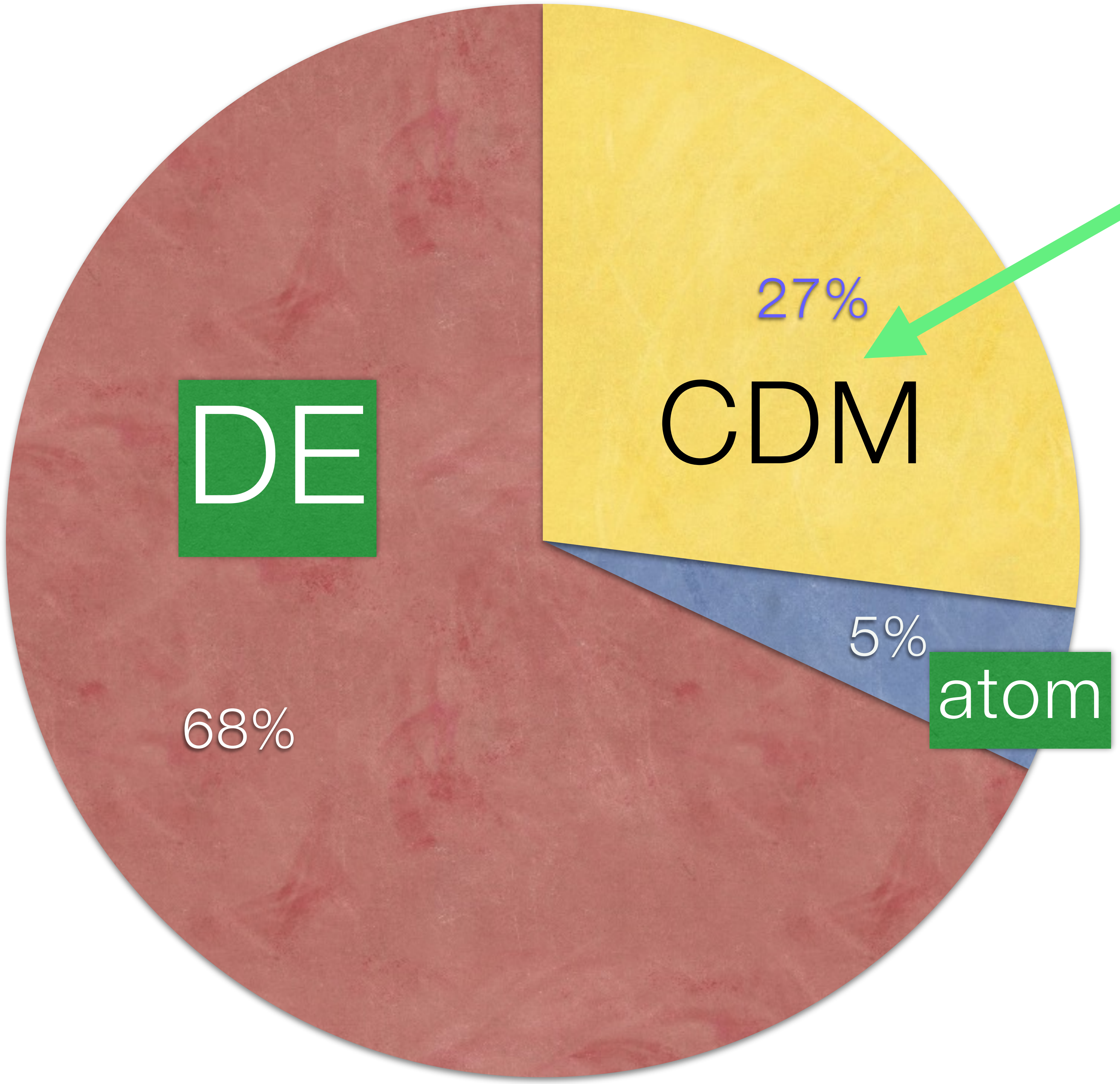

27%
CDM
DE
5%
atom
68%



Detection of “invisible” axion CDM by cavity detectors: CAPP, Yale, ADMX, etc.

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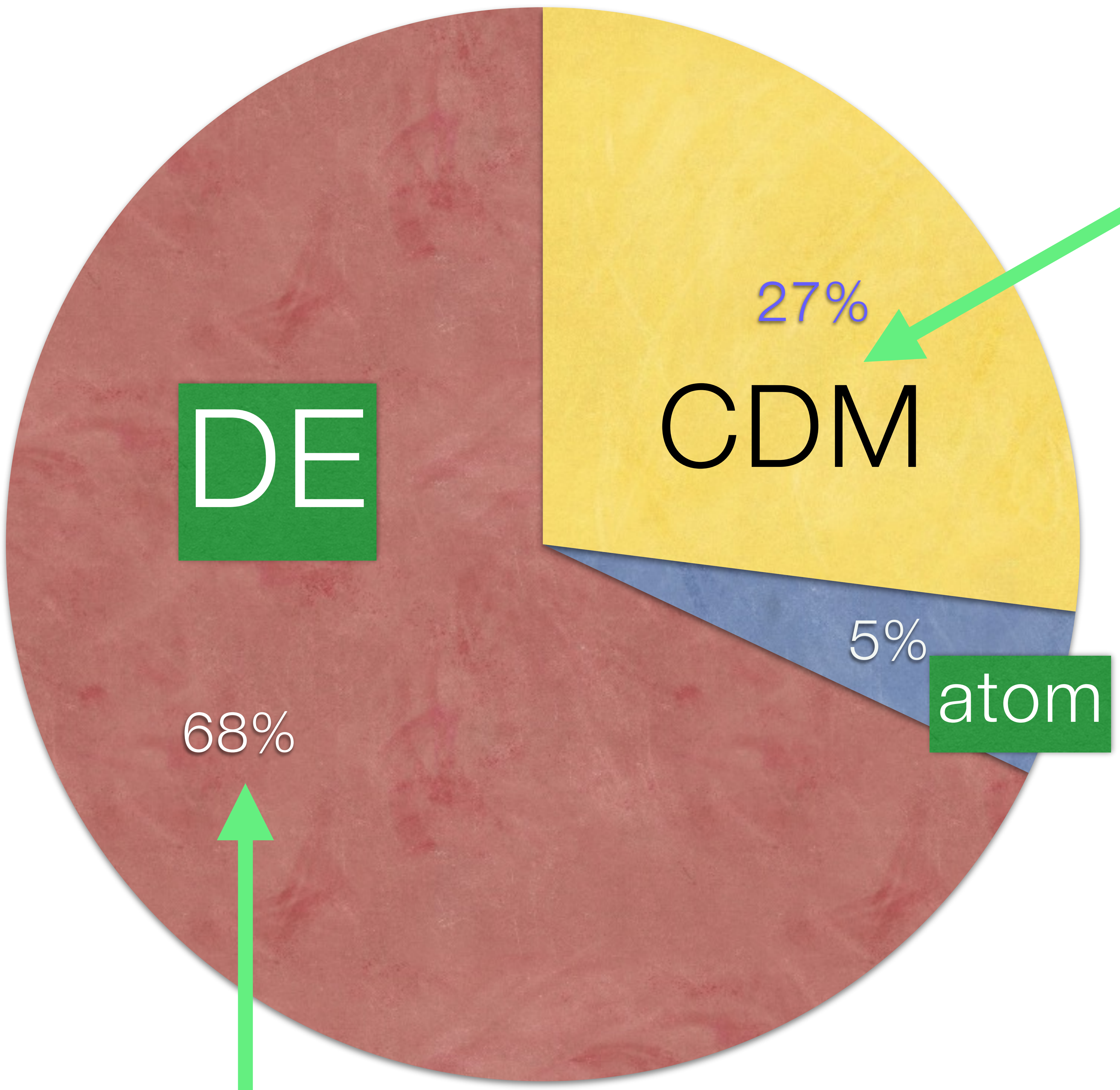

Quintessential axion

6. Conclusion

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1. Introduction

6. Conclusion

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2. “Invisible” axions

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