International workshop on
"Axion physics and dark matter cosmology"
@ Osaka University, December 20-21, 2017

# Cosmological Problems of QCD Axion

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Mainly Based on

- MK, Saikawa, Sekiguchi, arXiv:1412.0789
- MK, Yanagida, Yoshino, arXiv:1305.5338
- Harigaya, Ibe, MK, Yanagida, arXiv:1507.00119
- MK, Sonomoto, arXiv:1710.07269
- MK, Sekiguchi, Yamaguchi, Yokoyama, in progress

# 1. Axion

- Axion is the Nambu-Goldstone boson associate with $U(1)_{PQ}$ breaking and can be identified with the phase of PQ scalar

$$\Phi = |\Phi| e^{i\theta} = (\eta + \varphi) e^{ia/\eta}$$

$\eta$ : breaking scale

- Axion acquires mass through QCD non-perturbative effect

$$m_a \simeq 0.6 \times 10^{-5} \text{eV} \left( \frac{F_a}{10^{12} \text{GeV}} \right)^{-1}$$

$F_a = \eta / N_{\rm DW}$

$N_{\rm DW}$: domain wall number

- Axion is a good candidate for dark matter of the universe
- Cosmological evolution of axion (PQ scalar)
  - PQ symmetry breaking after inflation

    Formation of topological defects → Domain wall problem
  - PQ symmetry breaking before inflation

    Isocurvature perturbations → Isocurvature problem

# Today's Talk

- Introduction
- PQ symmetry breaking after inflation
  - Cosmological evolution of axion
  - Domain problem and comic axion density
- PQ symmetry breaking before inflation
  - Isocurvature perturbation problem
  - Suppression of Isocurvature Perturbations
- Conclusion

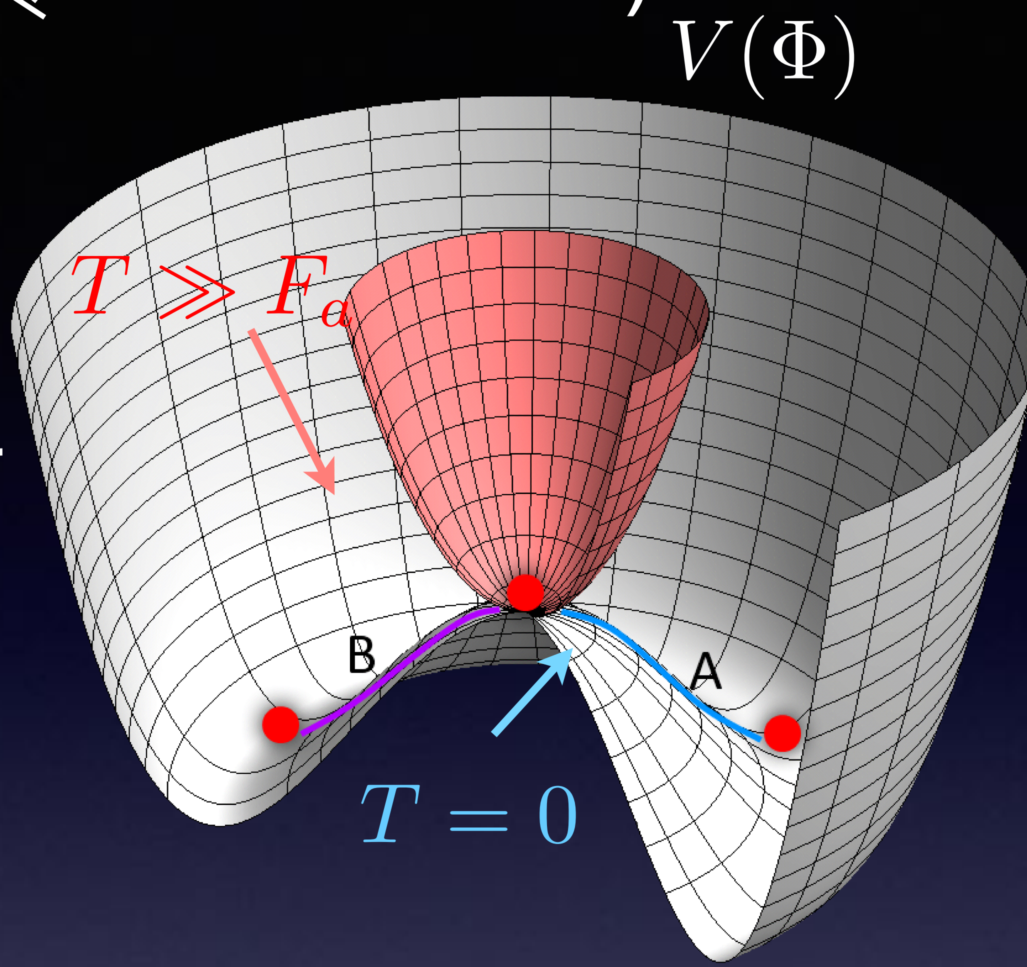
# 2. Cosmological Evolution of Axion (PQ after inflation)

$T \simeq \eta$

- $U_{PQ}(1)$ symmetry is broken
  - Axion is a phase direction of PQ scalar and massless

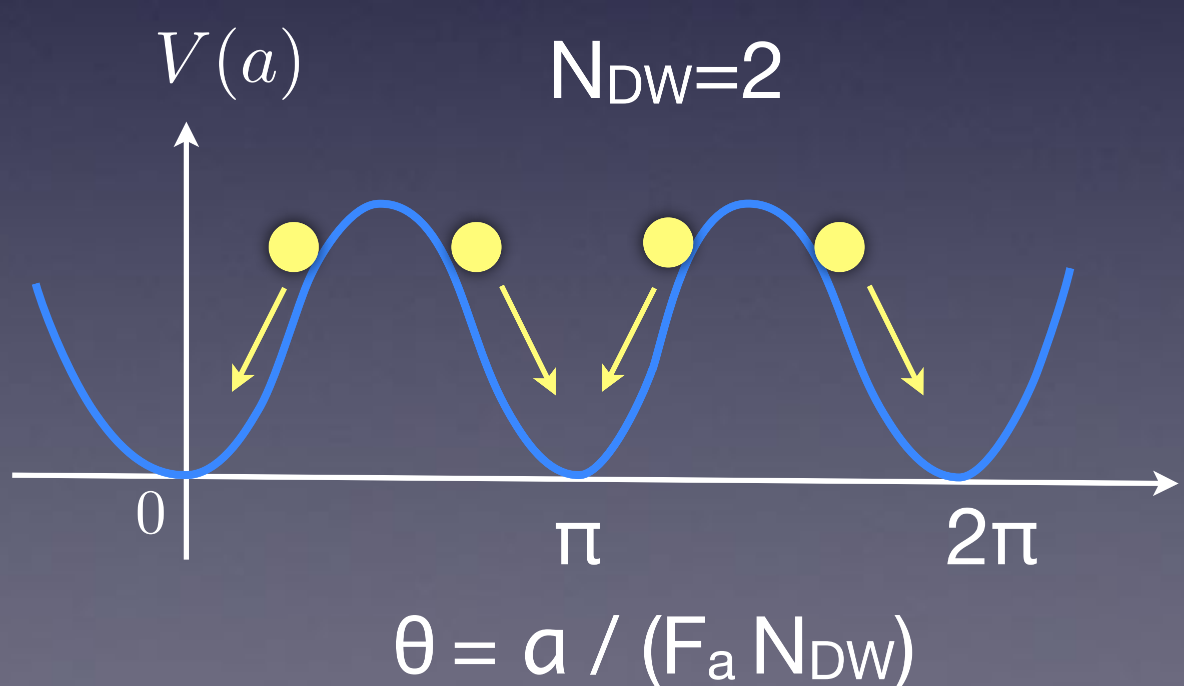
$$\Phi = |\Phi| e^{i\theta} = |\Phi| e^{ia/\eta} \qquad m_a = 0$$

  - Formation of Cosmic Strings



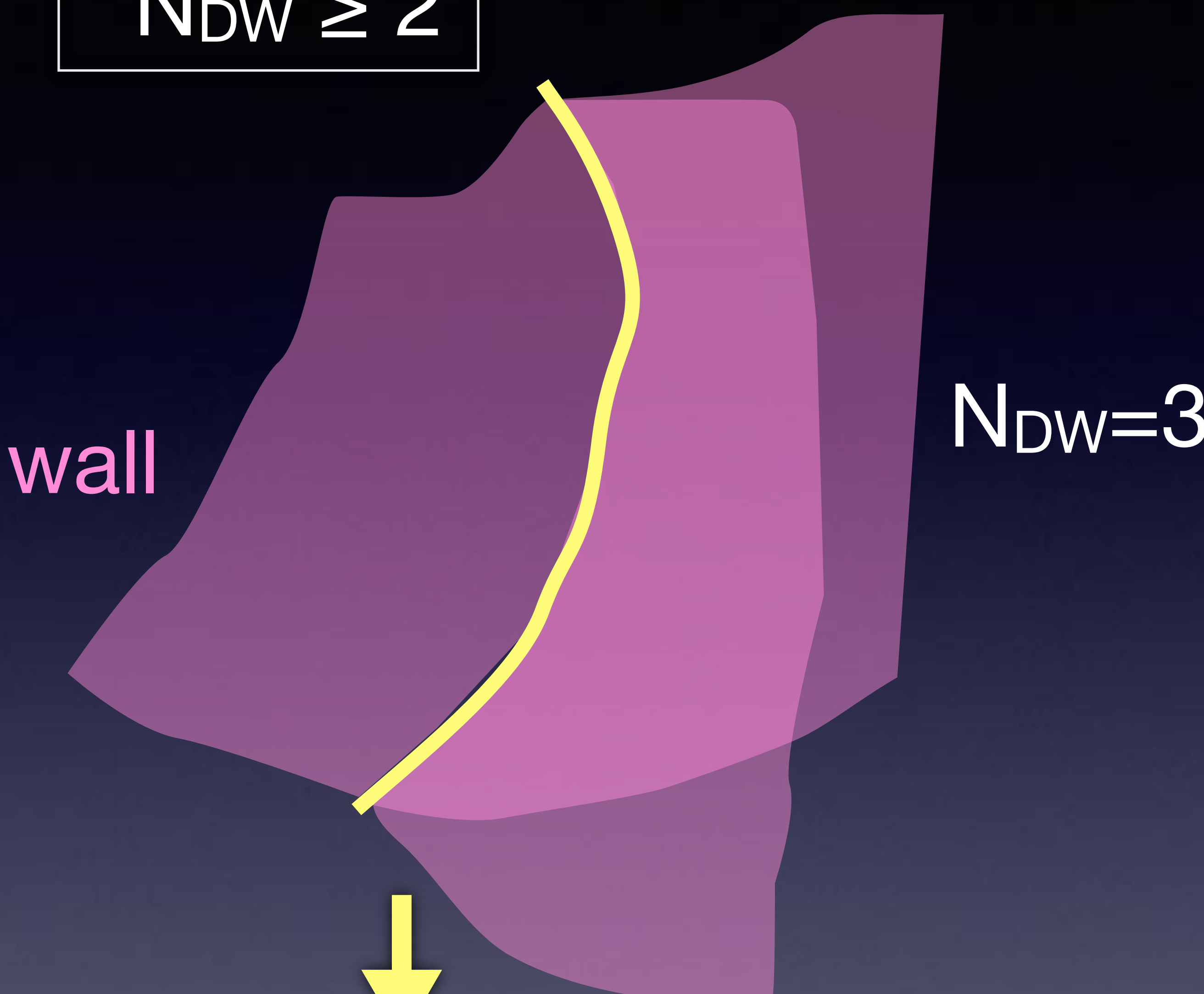

$T \simeq \Lambda_{QCD}$

- Axion acquires mass through non-perturbative effect
  - $U_{PQ}(1)$ is broken to $Z_{N_{DW}}$
  - Coherent oscillation
  - Formation of Domain Walls



- Domain walls attach to strings

$N_{DW} \geq 2$

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Stable and dominate the universe

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Domain Wall Problem

$N_{DW}=1$


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Collapse

Virenkin Everett (1982)
Barr Choi Kim (1986)

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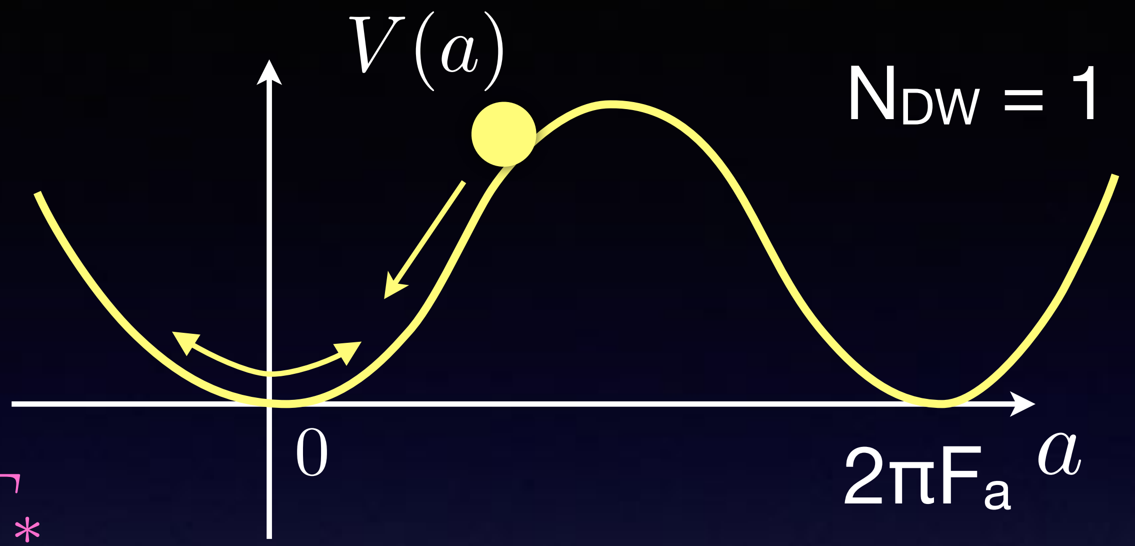
Axion overproduction

# 3. Cosmic Axion Density

## 3.1 Coherent axion oscillation

$$H \simeq m_a(T_*)$$




- Axion field starts to oscillate at $T = T_*$
- Coherent oscillation of axion field gives a significant contribution to the cosmic density ( $\Omega_{\rm CDM} h^2 \simeq 0.12$ )

$$\Omega_{a,\rm osc} h^2 \simeq 7 \times 10^{-4} \langle \theta_*^2 \rangle \left( \frac{F_a}{10^{10}\,\rm GeV} \right)^{1.19}$$

spatial average

$\langle \theta_*^2 \rangle \simeq 6$

$\theta_* = a_*/F_a$ : misalighnment angle at $T_*$

including anharmonic effect

$$\Omega_{a,\rm osc} h^2 \simeq 0.12 \quad \text{if} \quad F_a \simeq 2 \times 10^{11}\ \rm GeV$$

# 3.2 Axions from strings

- Axionic strings are produced when U(1) $_{PQ}$ symmetry is spontaneously broken

- Numerical Simulation

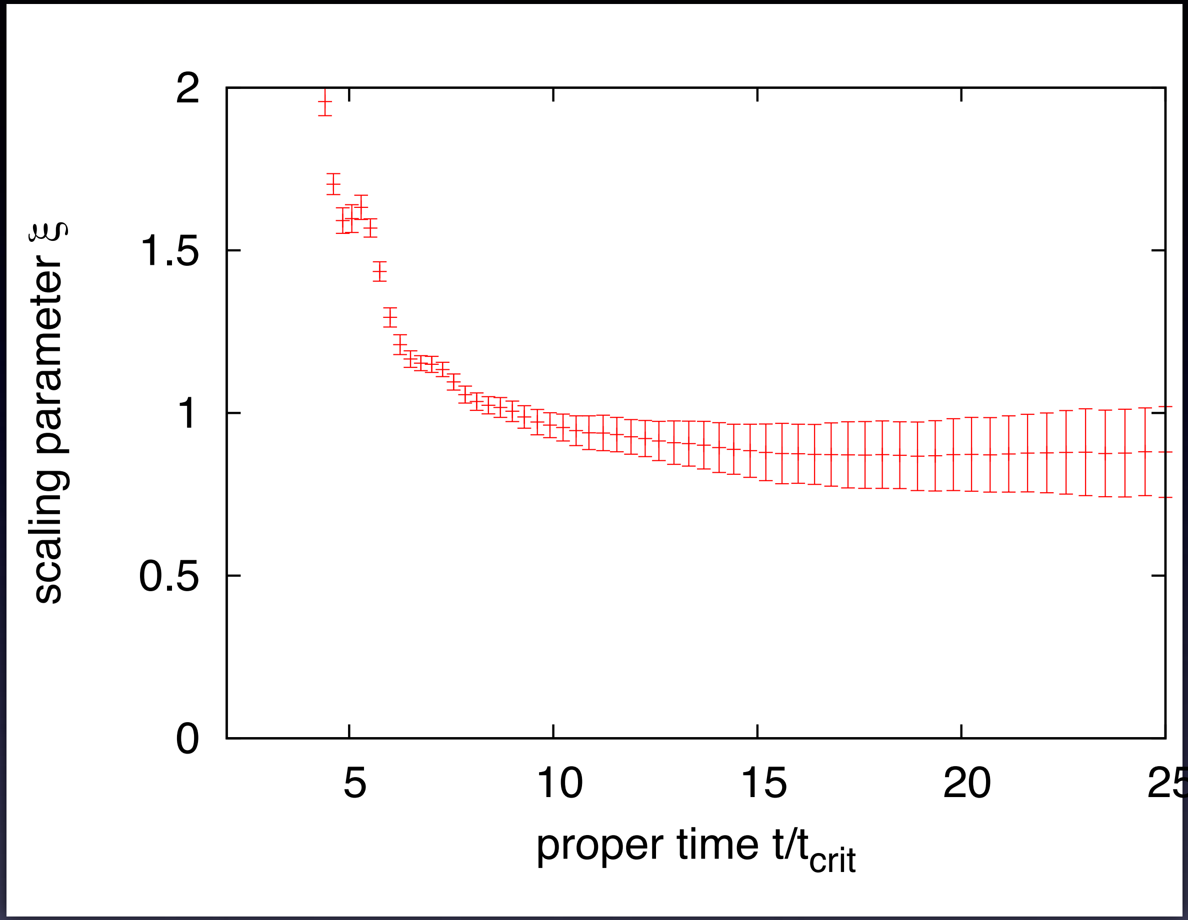
  Hiramatsu, MK, Sekiguchi, Yamaguchi, Yokoyama (2010)

  MK, Saikawa, Sekiguchi (2014)

- String network obeys scaling solution

$$\rho_{\text{string}} = \xi \frac{\mu}{t^2} \quad (\mu \sim \eta^2 : \text{string tension})$$

$$\xi = 1.0 \pm 0.5$$




- Scaling solution is established by emitting axions

- Emitted axion energy $\rho_{a,\,\text{str}}$ is estimated from $\rho_{\text{string}}$

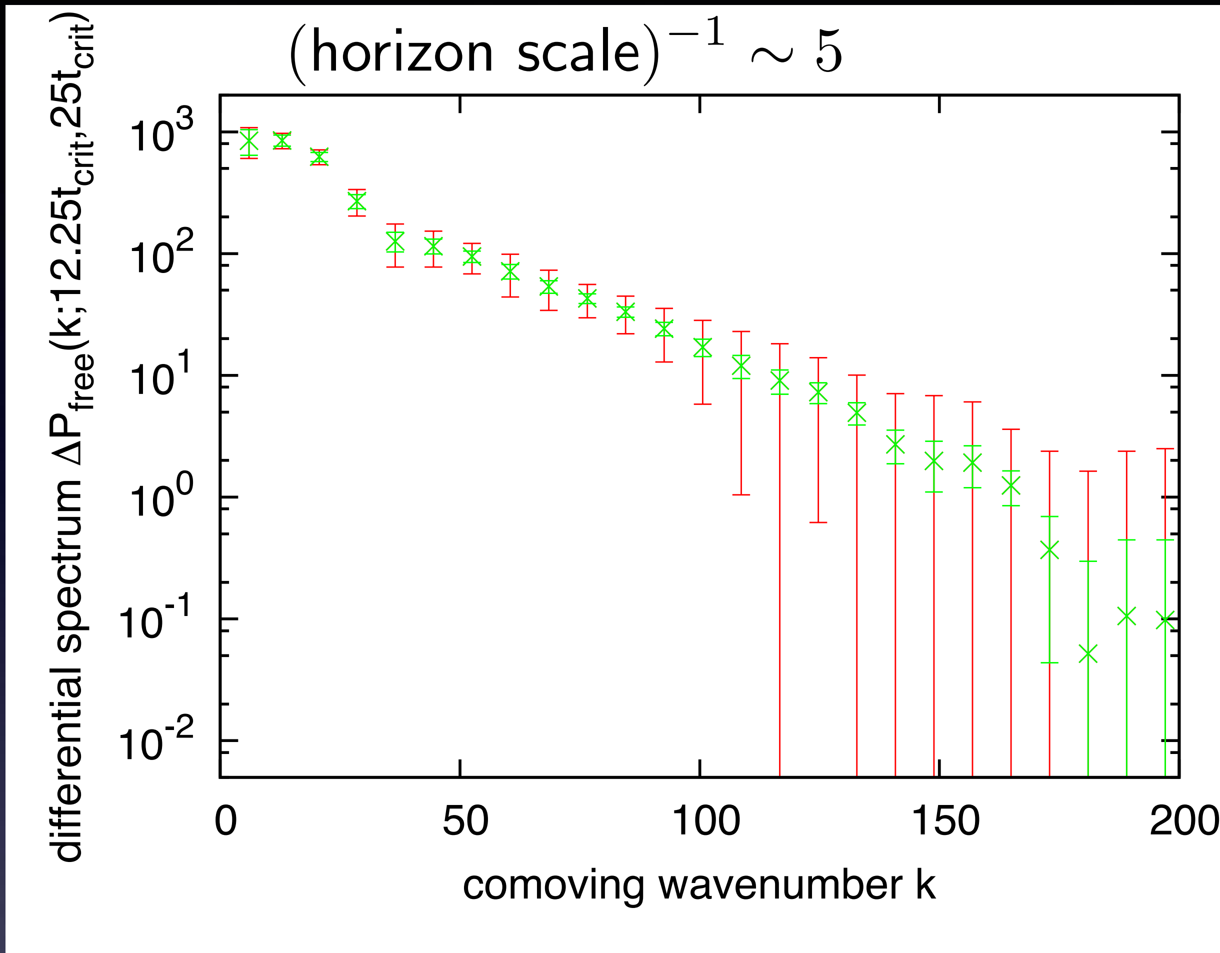
- If we know average energy $\bar{\omega}_a$ the axion density when it becomes non-relativistic is estimated as $\rho_a = m_a(\rho_{a,\text{str}}/\bar{\omega}_a)$

# Density of Axions from Strings

- Energy Spectrum

  peaked at k ~ horizon scale

  exponentially suppressed at higher k




- Average energy

$$\bar{\omega}_a = \epsilon \frac{2\pi}{t}$$

$$\epsilon = 4.02 \pm 0.70$$

MK, Saikawa, Sekiguchi (2014)

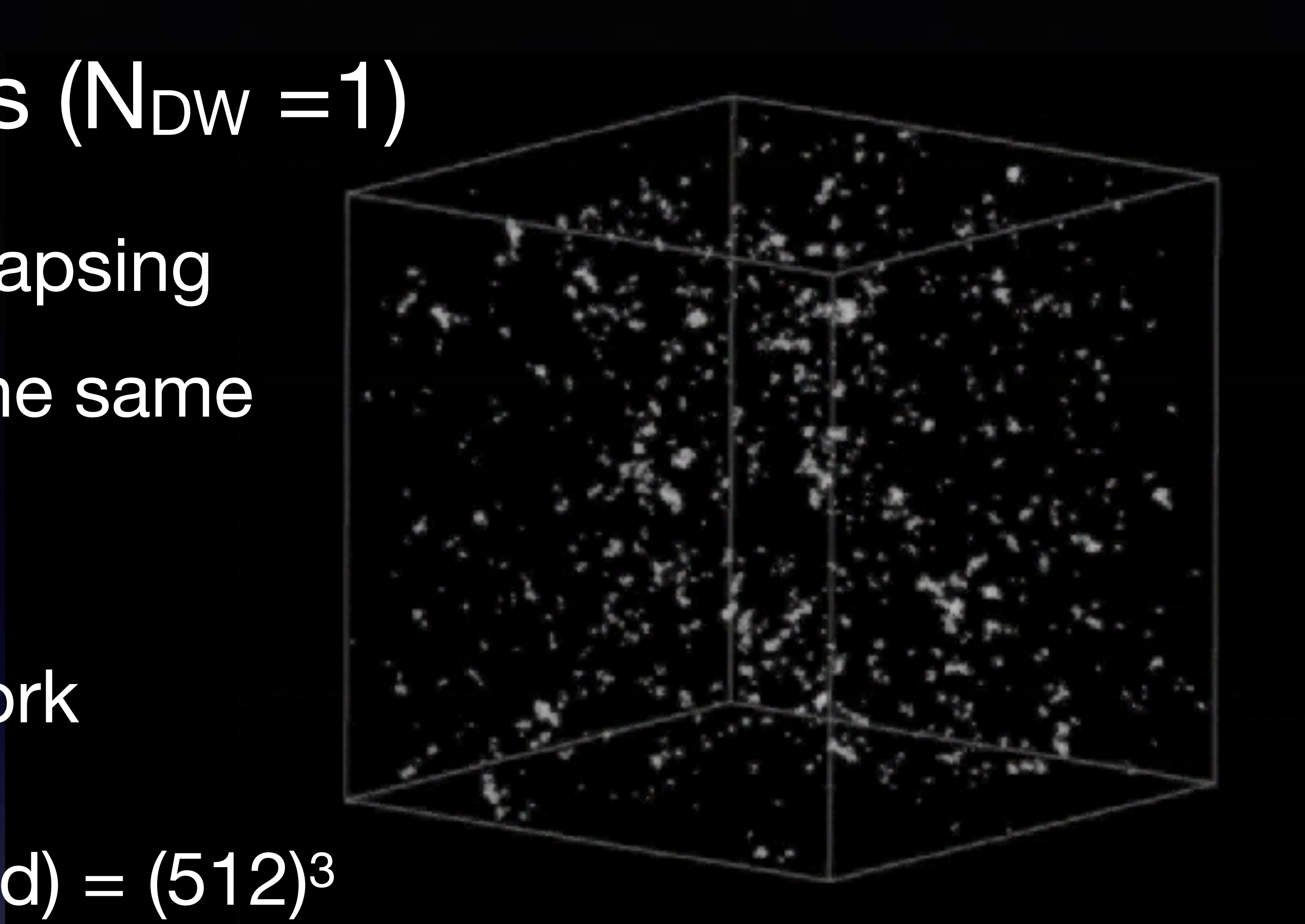
- Cosmic density of produced axion

$$\Omega_{a,\text{string}} h^2 = (7.3 \pm 3.9) \times 10^{-3} N_{\text{DW}}^2 \left( \frac{F_a}{10^{10}\,\text{GeV}} \right)^{1.19}$$

$$\Omega_{a,\text{osc}} h^2 \simeq 4 \times 10^{-3} \left( \frac{F_a}{10^{10}\,\text{GeV}} \right)^{1.19}$$

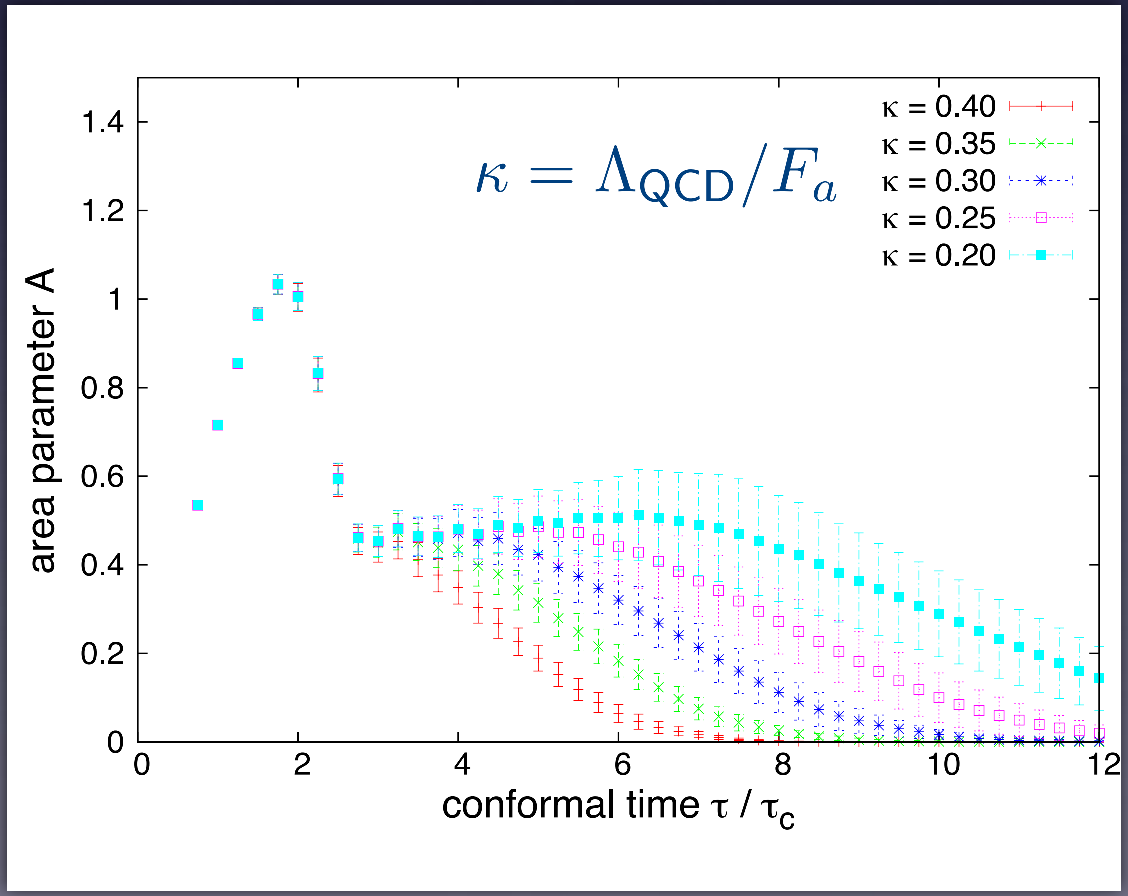
# 3.3 Axion from Domain Walls ($N_{DW}=1$)

- Axion energy density from collapsing domain walls is estimated in the same way as strings



- Simulation of string-wall network
  - Lattice simulation with N(grid) = $(512)^3$
  - Scaling property
  - Average energy

Hiramatsu, MK, Saikawa, Sekiguchi (2012)




- Axions from collapsed domain walls

$$\Omega_{a,\rm wall}h^2 = (5.4\pm 2.1)\times 10^{-3} \times \left(\frac{F_a}{10^{10}\rm GeV}\right)^{1.19}$$

# Cosmic Axion Density ($N_{DW}$ =1)

- Total cosmic axion density

$$\Omega_{a,\mathrm{tot}}h^2 = \Omega_{a,\mathrm{osc}}h^2 + \Omega_{a,\mathrm{string}}h^2 + \Omega_{a,\mathrm{wall}}h^2$$
$$= (1.7 \pm 0.4) \times 10^{-2} \left(\frac{F_a}{10^{10}\mathrm{GeV}}\right)^{1.19}$$

$\longleftrightarrow$ $\Omega_{\mathrm{CDM}}h^2 \simeq 0.12$

- Constraint on $F_a$

$$F_a \lesssim (4.2 - 6.5) \times 10^{10}\ \mathrm{GeV}$$

$$m_a \gtrsim (0.9 - 1.4) \times 10^{-4}\ \mathrm{eV}$$

# 3.4 Axion from Walls ($N_{DW} \geq 2$)

- Wall-string networks are stable and soon dominate the universe

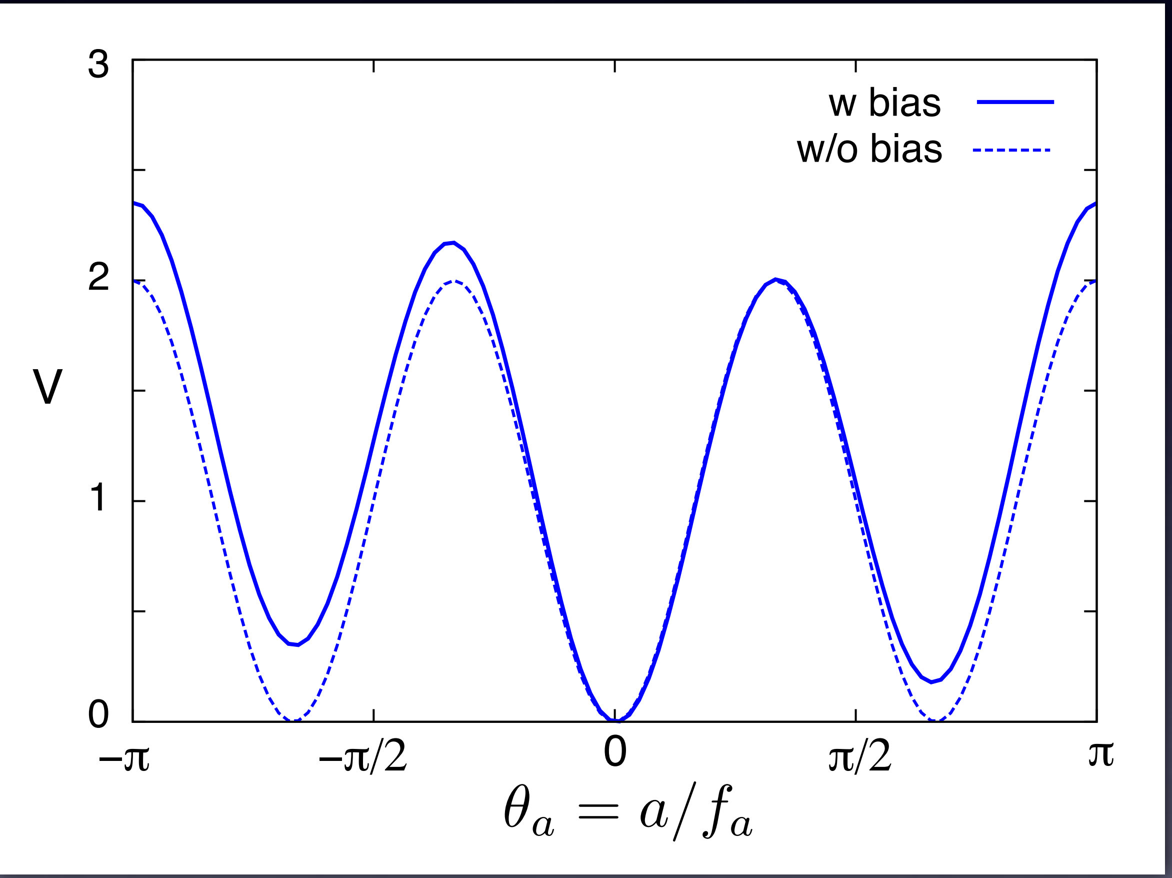
  → Domain Wall Problem

- The problem can be avoided by introducing a "bias" term which explicitly breaks PQ symmetry

$$V_{\text{bias}} = -\Xi \eta^3 \left( \Phi e^{-i\delta} + \text{h.c.} \right)$$

$\Xi$ : bias parameter  Sikivie (1982)

$\delta$ : phase of bias term



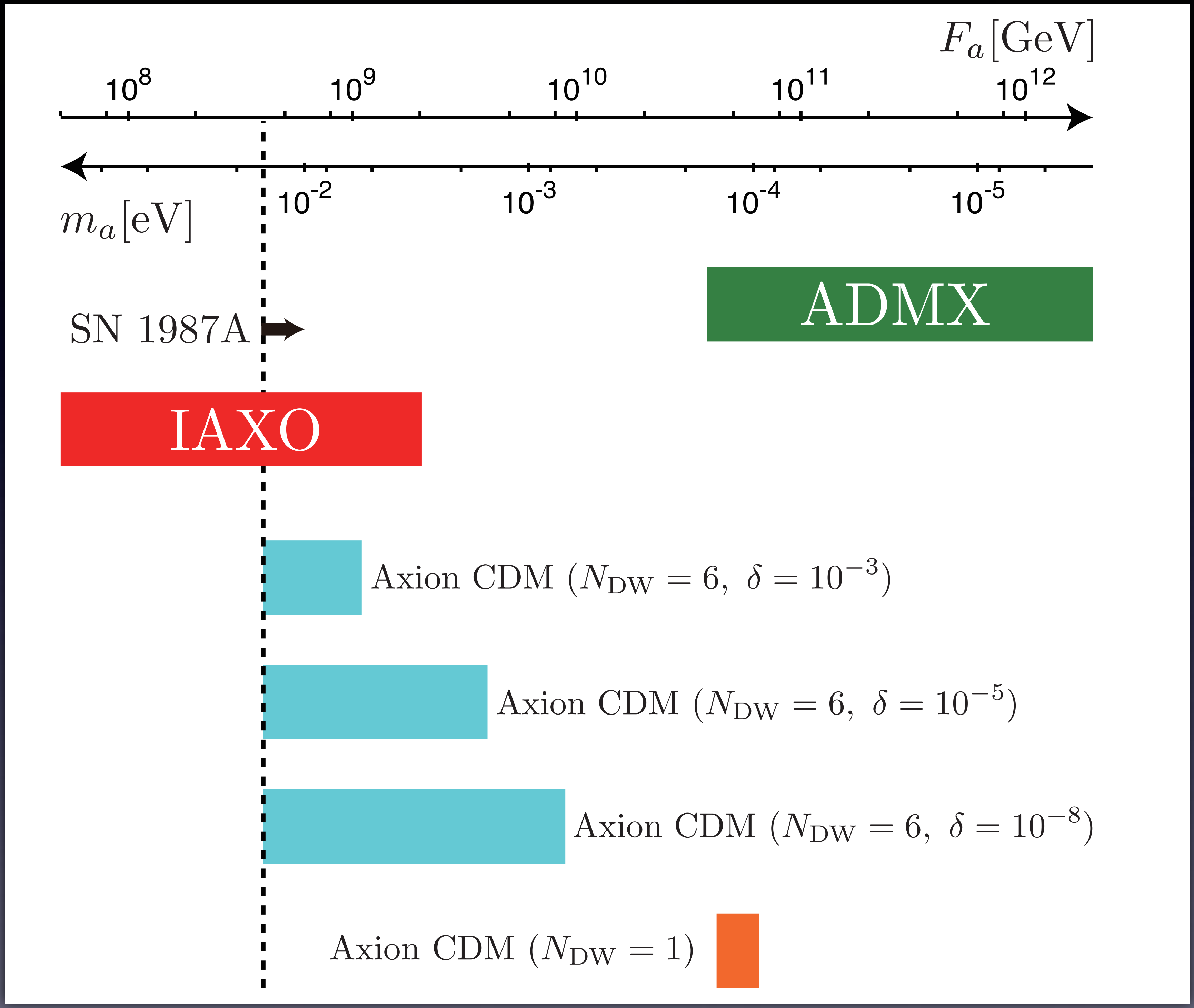

- Bias term lifts degenerated vacua and leads to DW annihilation

  → large bias is favored

- Bias term shifts the minimum of the potential $\langle \theta \rangle \neq 0$ and spoils the original idea of Peccei and Quinn → small bias is favored

  → More stringent constraint on $F_a$
  Axion can be dark matter for smaller $F_a$

# 3.5 Summary: case of symmetry breaking after inflation



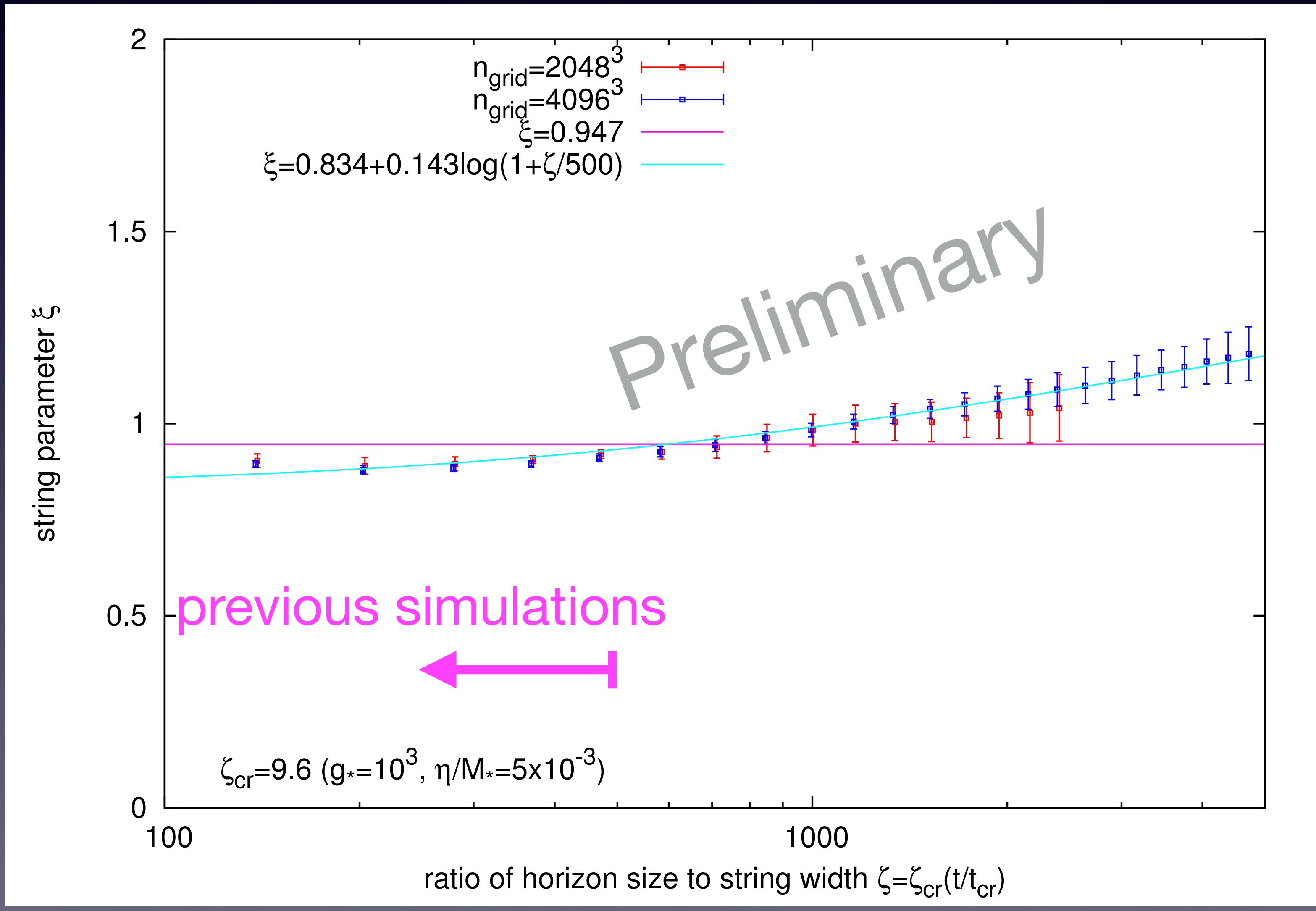

parameter ranges where axion can be dark matter

- Axion can be dark matter of the universe for $F_a$ ~ $10^9$ GeV or ~$5\text{X}10^{10}$ GeV and can be probed in the next generation experiments

# 3.6 Recent Progress

MK, Sekiguchi, Yamaguchi, Yokoyama in progress

- Our prediction heavily depends on scaling behavior
- We need know the scaling behavior on longer time scale
- We are updating our simulations ( from $N_{grid}=512^3$ to $4096^3$ )




- New simulation suggests scaling parameter ξ slightly increases at large t

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- The previous result might have larger systematic errors

# 4. Axion in the Inflationary Universe (~~PQ~~ before inflation)

- If PQ symmetry is broken during or before inflation
  - Strings and domain walls are diluted away by inflation
    No domain wall problem
  - Only coherent oscillation gives a significant contribution to the cosmic density

$$\Omega_{a,\mathrm{osc}} \simeq 0.19\ \theta_*^2 \left( \frac{F_a}{10^{12}\mathrm{GeV}} \right)^{1.19}$$

    Inflation makes $\theta_*$ the same in the whole observable universe ( $\theta_*$ is a kind of free parameter )

  - Isocuravture perturbation problem

## 4.1 Axion Isocurvature Fluctuations

- Axion acquires fluctuations during inflation

$$\delta a = F_a \delta\theta_a \simeq \frac{H_{\mathsf{inf}}}{2\pi} \quad \Leftarrow \quad \langle \delta a^2 \rangle = F_a^2 \langle \delta\theta_a^2 \rangle = (H_{\mathsf{inf}}/2\pi)^2$$

$$\rho_a = \rho_a(t) + \delta\rho_a(t,\vec{x}) = m_a^2 [a(t) + \delta a(t,\vec{x})]^2/2$$

$$= m_a^2 F_a^2 [\theta_* + \delta\theta_a(\vec{x})]^2/2 \quad \Leftarrow \quad a = F_a \theta_a$$

- For small fluctuation

$$\theta_* > \delta\theta_a \quad \Rightarrow \quad \frac{\delta\rho_a}{\rho_a} \simeq 2\frac{\delta\theta_a}{\theta_*}$$

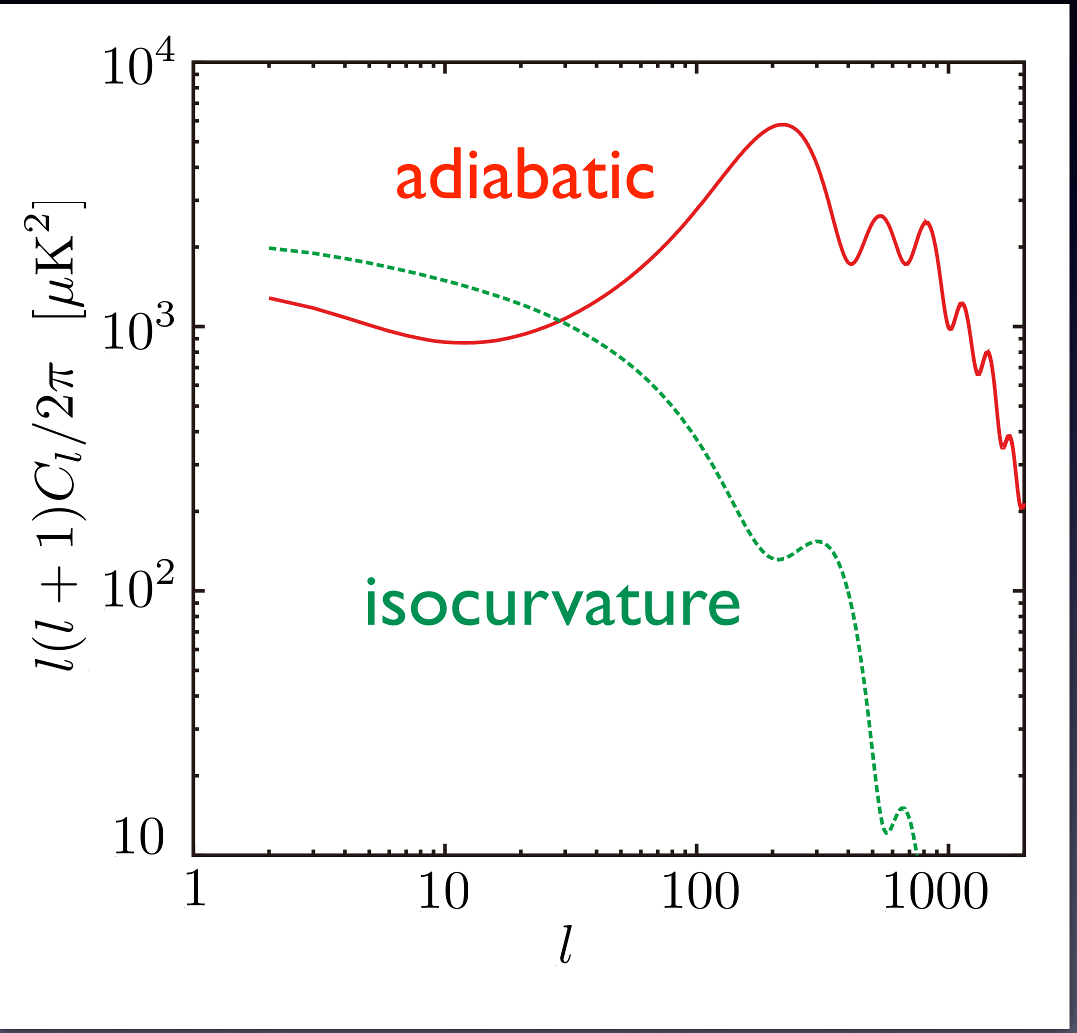
# 4.1 Axion Isocurvature Fluctuations

- Axion fluctuations contribute to CDM isocurvature density perturbation

$$S = \frac{\delta\rho_{\rm CDM}}{\rho_{\rm CDM}} - \frac{3\delta\rho_\gamma}{\rho_\gamma} = \frac{\Omega_a}{\Omega_{\rm CDM}}\frac{\delta\rho_a}{\rho_a}$$

- Isocurvature perturbations lead to CMB angular power spectrum which is different from adiabatic one

CMB angular Power spectrum




- Stringent constraint on amplitude of isocurvature perturbation

$$\beta_{\rm iso} \equiv \frac{P_S(k_0)}{P_\zeta(k_0) + P_S(k_0)}$$

$k_0 = 0.002\ {\rm Mpc}^{-1}$

- WMAP9

$$\beta_{\rm iso} < 0.047\ (95\%\ {\rm CL})$$

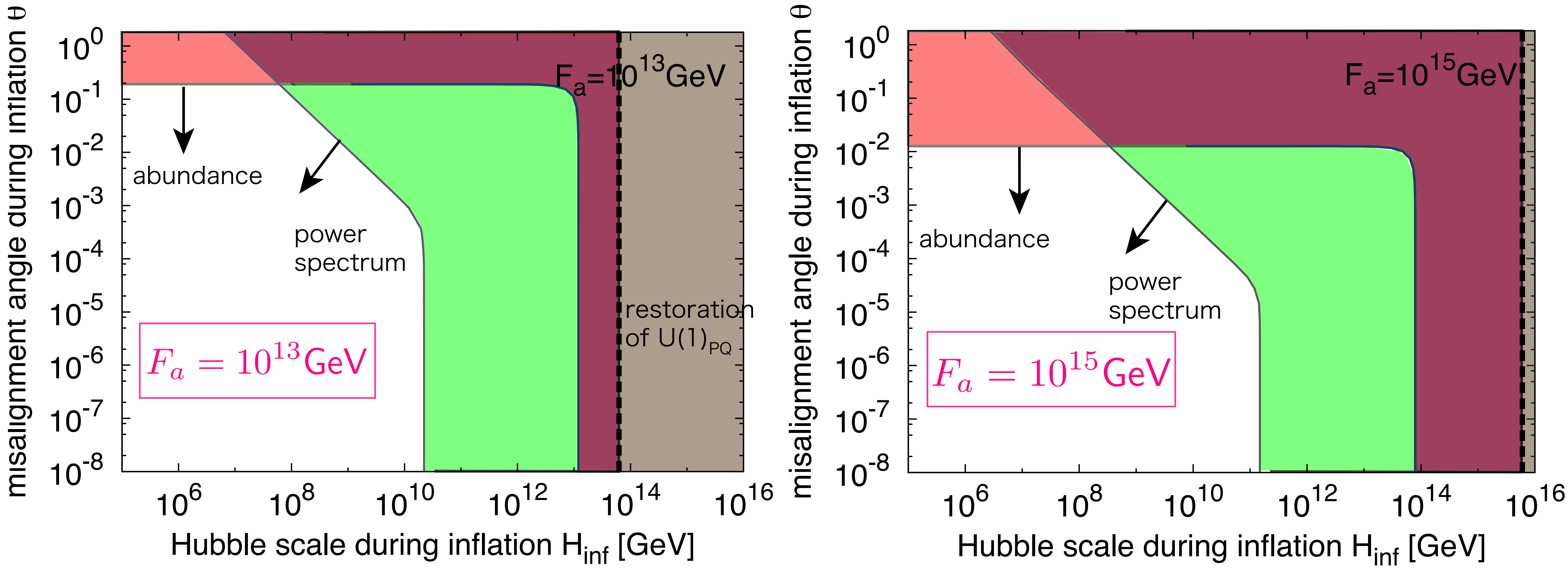
- PLANCK 2015

$$\beta_{\rm iso} < 0.033\ (95\%\ {\rm CL})$$

# Axion isocurvature fluctuations

- Stringent constraints from CMB

Hikage, MK, Sekiguchi, T.Takahashi (2012)




Constraint from power spectrum is updated including Planck data

- Only low energy scale inflation models are allowed
  High scale inflation ( $H_{inf} > 10^{13}$GeV ) inconsistent with axion

- If axion is dark matter

$$H_{\rm inf} < 2.2 \times 10^7 {\rm GeV} \left( \frac{F_a}{10^{12}\ {\rm GeV}} \right)^{0.41}$$

# High scale inflation

- Observationally high scale inflation is favored because it is testable by observing B-mode polarization of CMB

➡ Tensor mode (gravitational wave) produced during inflation

r : tensor-to-scalar ratio

$$H_{\mathrm{inf}} = 8.6 \times 10^{13}\ \mathrm{GeV}\,(r/0.1)^{1/2}$$

r ~ 0.01 by experiments on the earth

r ~ 0.001 by satellite experiments

# 4.2 Suppressing Isocurvature Perturbations

- Amplitude of isocurvature perturbations is determined by fluctuations of misalignment angle

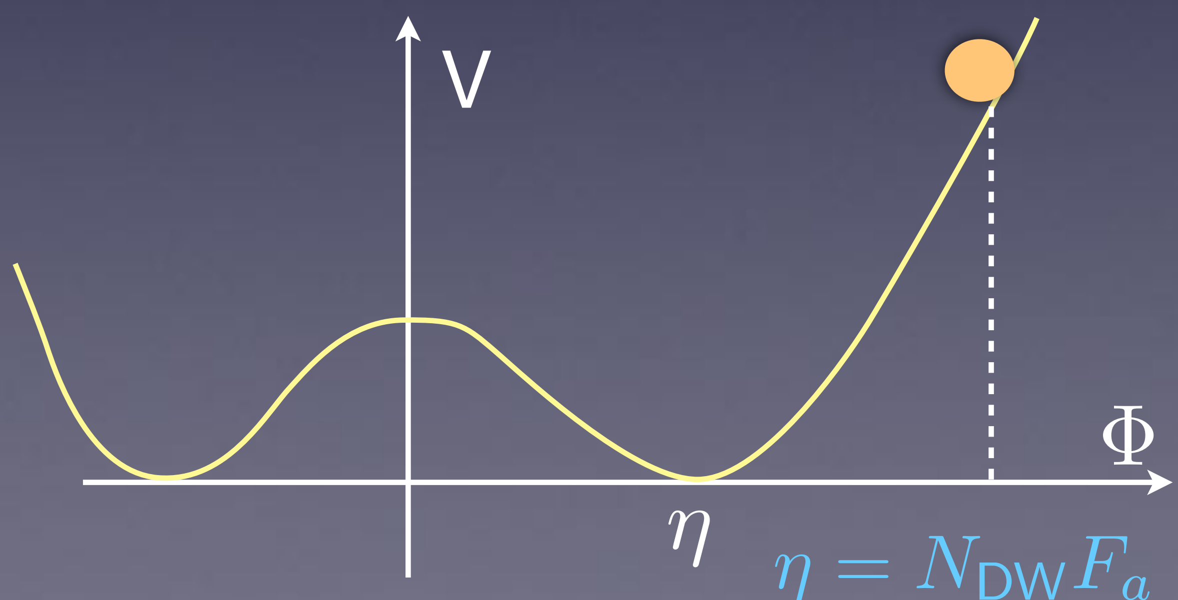
$$\delta\theta_a \simeq \frac{N_{\rm DW}}{\eta}\left(\frac{H_{\rm inf}}{2\pi}\right) = \frac{1}{F_a}\left(\frac{H_{\rm inf}}{2\pi}\right)$$

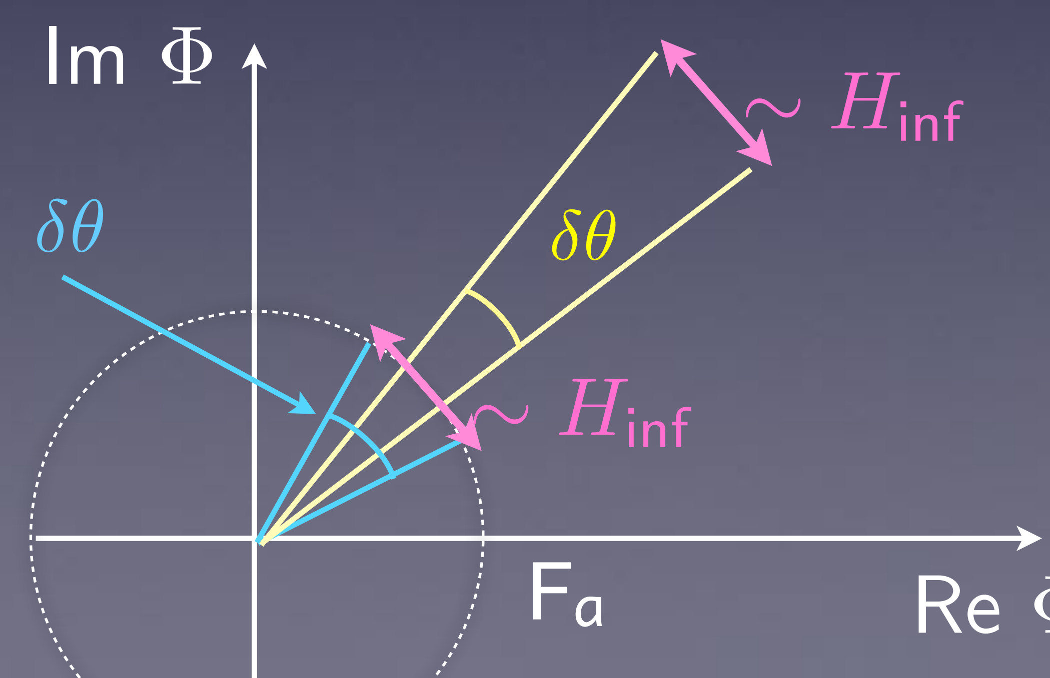
$$\frac{\delta\rho_a}{\rho_a} \simeq 2\frac{\delta\theta_a}{\theta_*}$$

- If PQ field has a large value during inflation effective PQ scale becomes large

$$\Rightarrow \quad \delta\theta_a \simeq \frac{N_{\rm DW}}{|\Phi|}\left(\frac{H_{\rm inf}}{2\pi}\right) \quad \Rightarrow \quad |\Phi| \gg F_a \Rightarrow \delta\theta_a \searrow$$

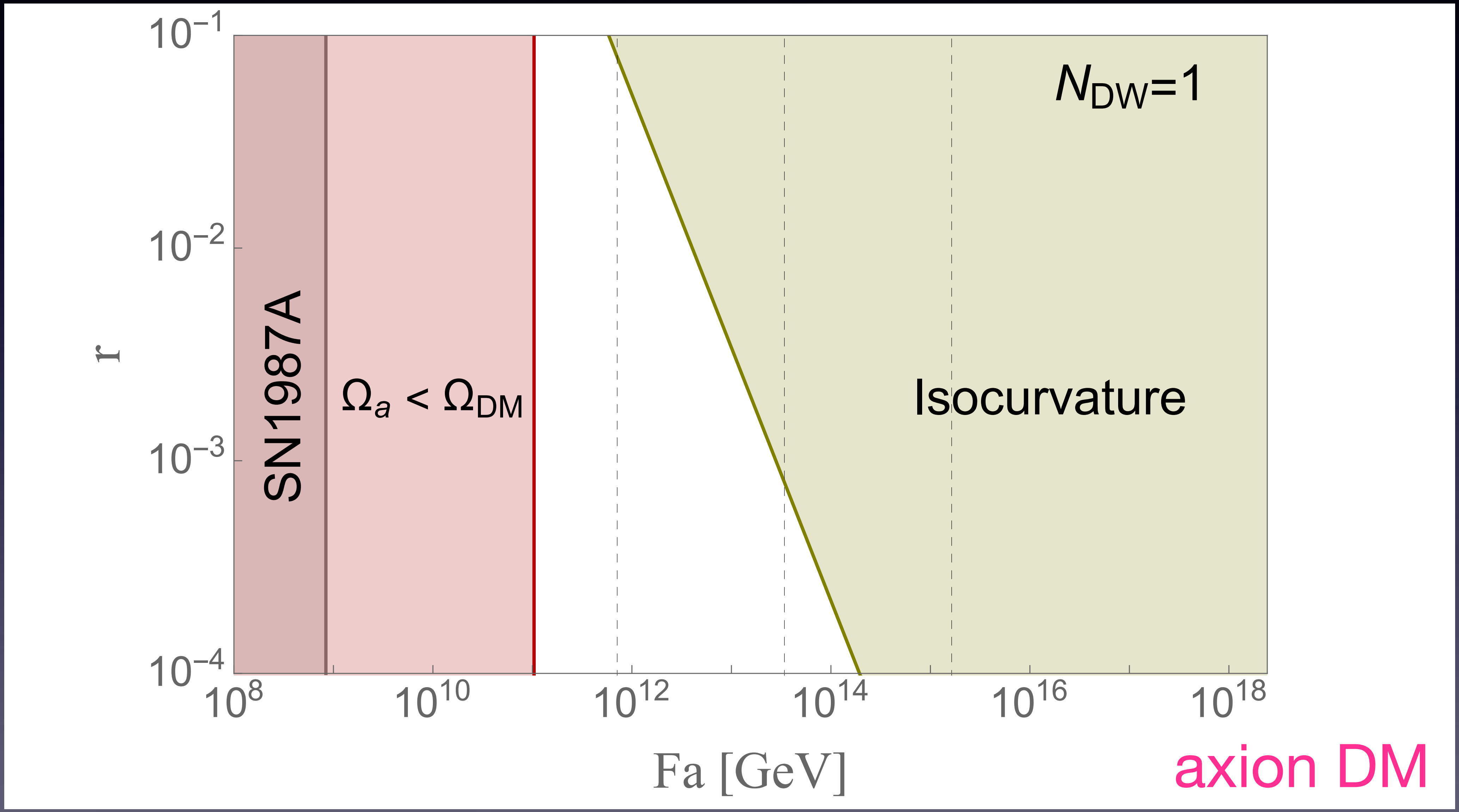
⇒ suppress isocurvature perturbations  Linde (1991)



- Assuming PQ field has a large field value $|\Phi| \simeq M_p$ during inflation and axion is dark matter

$$H_{\text{inf}} = 8.6 \times 10^{13}\,\text{GeV}\,(r/0.1)^{1/2}$$




- Dark matter axion is consistent with high scale inflation whose tensor mode is detectable in near future

- However, PQ field oscillates after inflation

→ Large fluctuations of PQ field through parametric resonance

This leads to non-thermal restoration of $U(1)_{PQ}$ symmetry

$$V_{PQ} \simeq \frac{\lambda}{2}(|\Phi|^2 - \eta^2)^2 + \lambda\langle|\delta\Phi|^2\rangle|\Phi|^2$$

$\langle|\delta\Phi|^2\rangle \gtrsim \eta^2$

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symmetry is restore

- Strings and domain walls are produced

- To avoid defect formation, PQ field must settle down to the minimum before the fluctuations fully develop

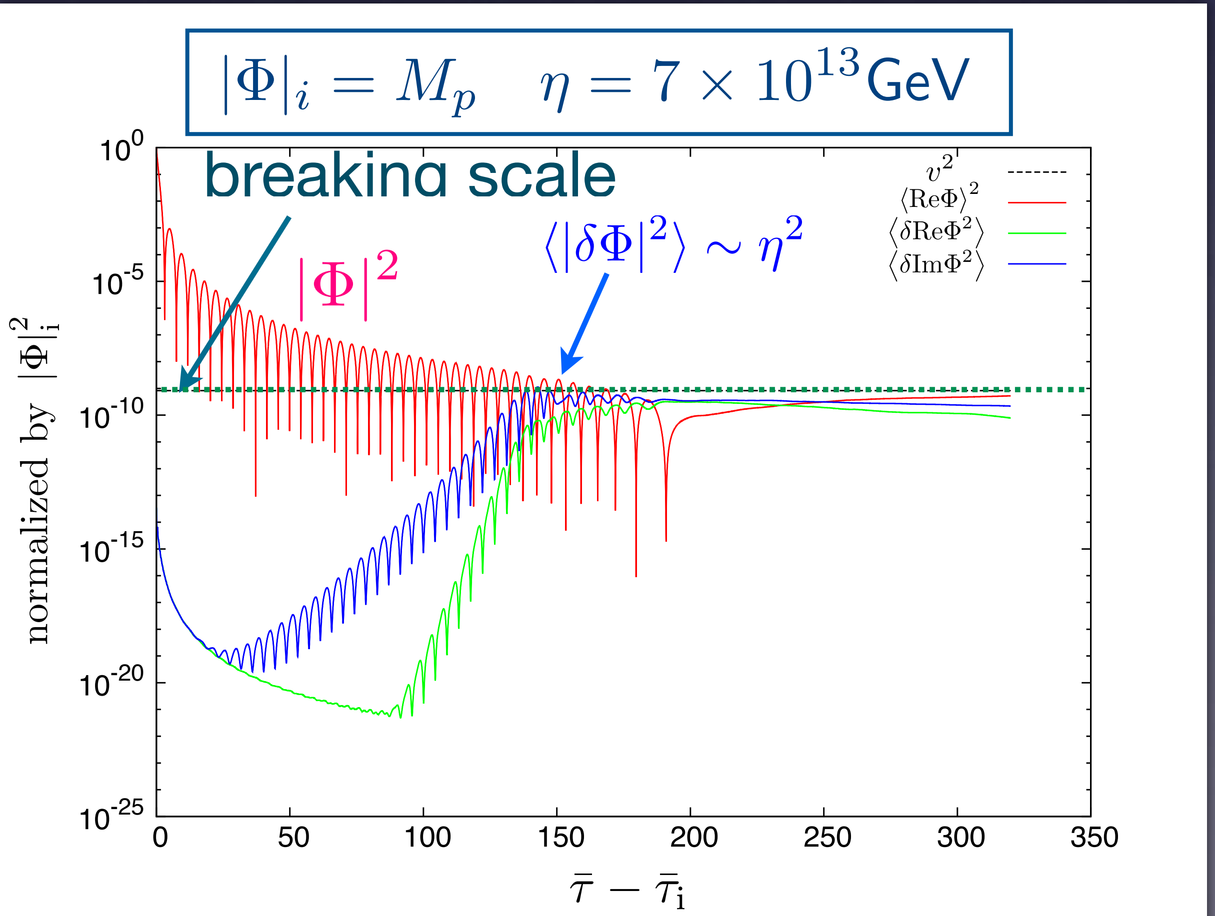
- Lattice simulation

MK, Yanagida, Yoshino (2013)

Lower bound on breaking scale η

→ $$\eta \gtrsim 10^{-4}|\Phi|_i$$

$\Phi_i$ initial value of PQ field



# Constraint in high scale inflation models

MK, Yanagida, Yoshino (2013)

- Assume axion is dark matter

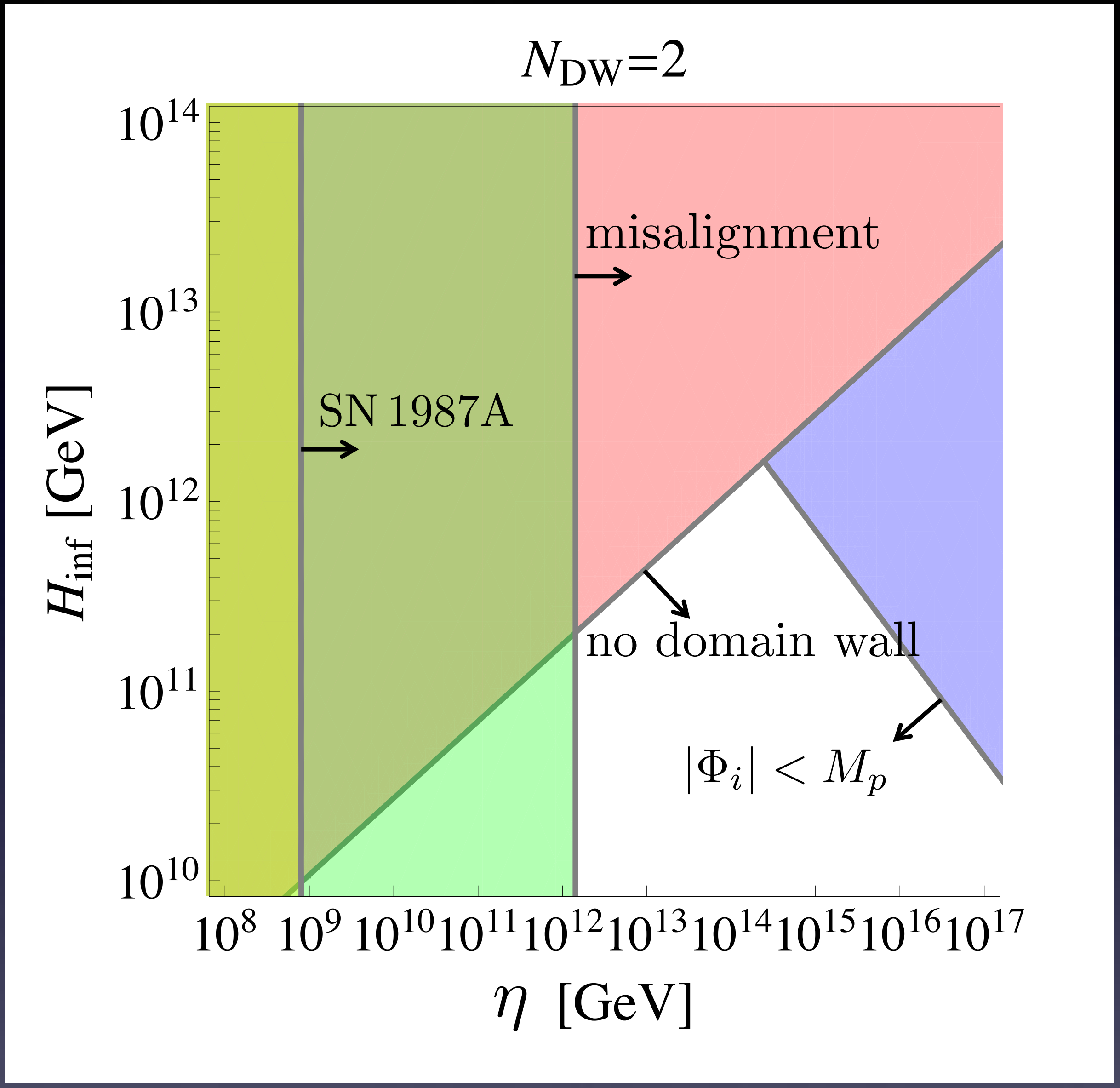
$$\Omega_a = \Omega_{\rm CDM}$$

- No domain wall and isocurvature perturbation problems

$$H_{\rm inf} \lesssim 10^{12}\ {\rm GeV}$$

$$(r \lesssim 5 \times 10^{-5})$$

- Too small for tensor mode to be detected




( If axion density < CDM density, high scale inflation ($H_{\rm inf} = 10^{13}$ GeV) consistent with axion for $\theta_a < O(10^{-(2\text{-}3)})$ )

## 4.3 Model without cosmological problems

- If PQ potential is controlled by $V_{\text{PQ}} \sim |\Phi|^{2n} \quad (n \geq 3)$
  - oscillation is very slow ➡ parametric resonance is ineffective
- Model

$$V(\Phi) = -m_\Phi^2|\Phi|^2 + \lambda_4^2|\Phi|^4 + \frac{\lambda_6^2}{M_p^2}|\Phi|^6 - c_H H^2|\Phi|^2$$

  - during inflation

    $|\Phi| \sim M_p$ due to $|\Phi|^6$ and Hubble induced terms

  - after inflation, PQ field oscillates slowly until its amplitude becomes

$$\Phi_{\text{osc-4}} = 10^{16}\text{GeV}\left(\frac{\lambda_4}{7\times 10^{-8}}\right)\left(\frac{2\times 10^{-5}}{\lambda_6}\right)$$

  - afterwards PQ field oscillates by quartic term
  - To avoid domain wall problem $\Phi_{\text{osc-4}} \lesssim 10^4\eta$

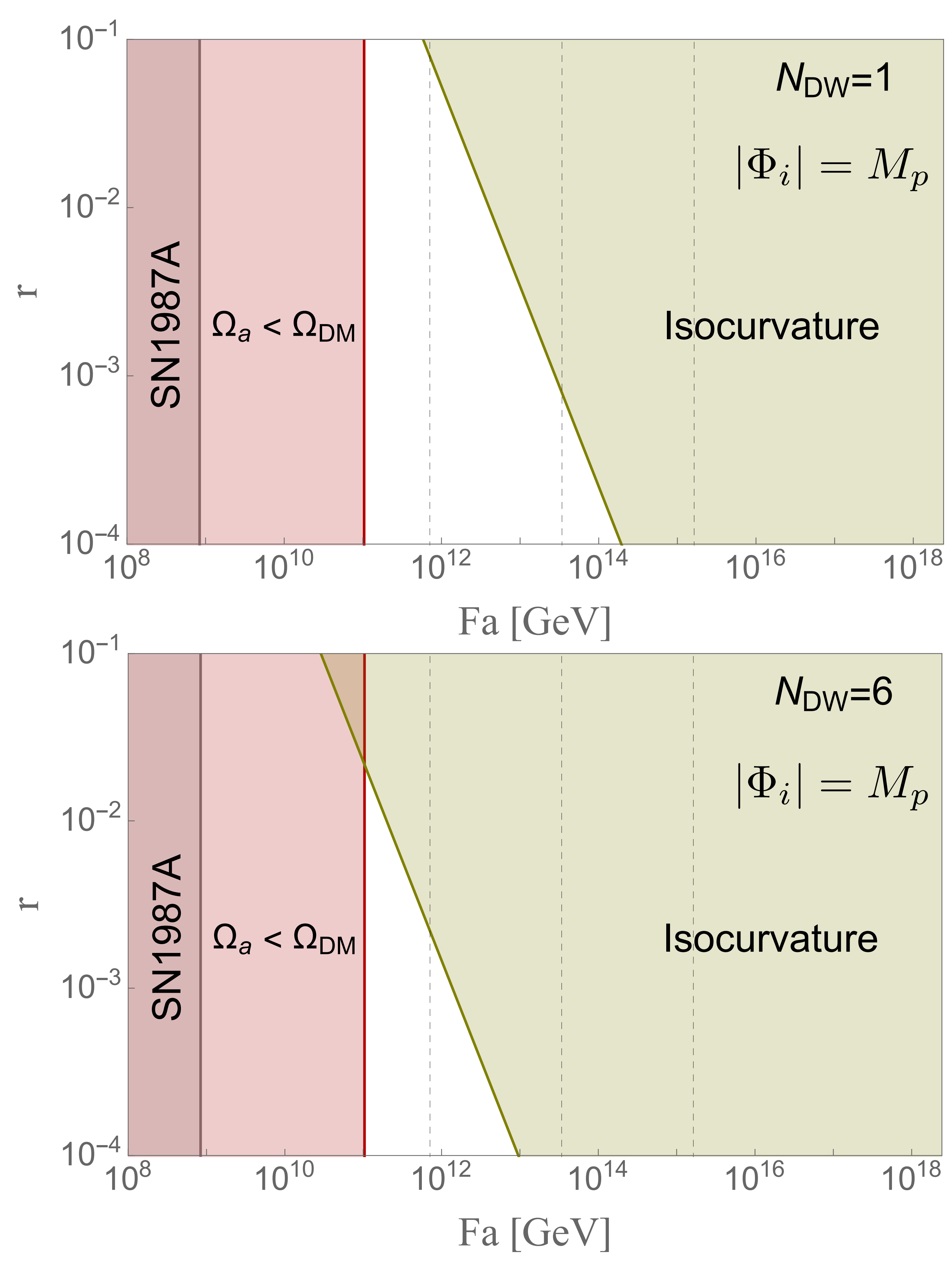
Axion dark matter is allowed by high scale inflation with

r < 0.1 for $N_{DW} = 1$

$H_{\rm inf} < 8 \times 10^{13}$ GeV

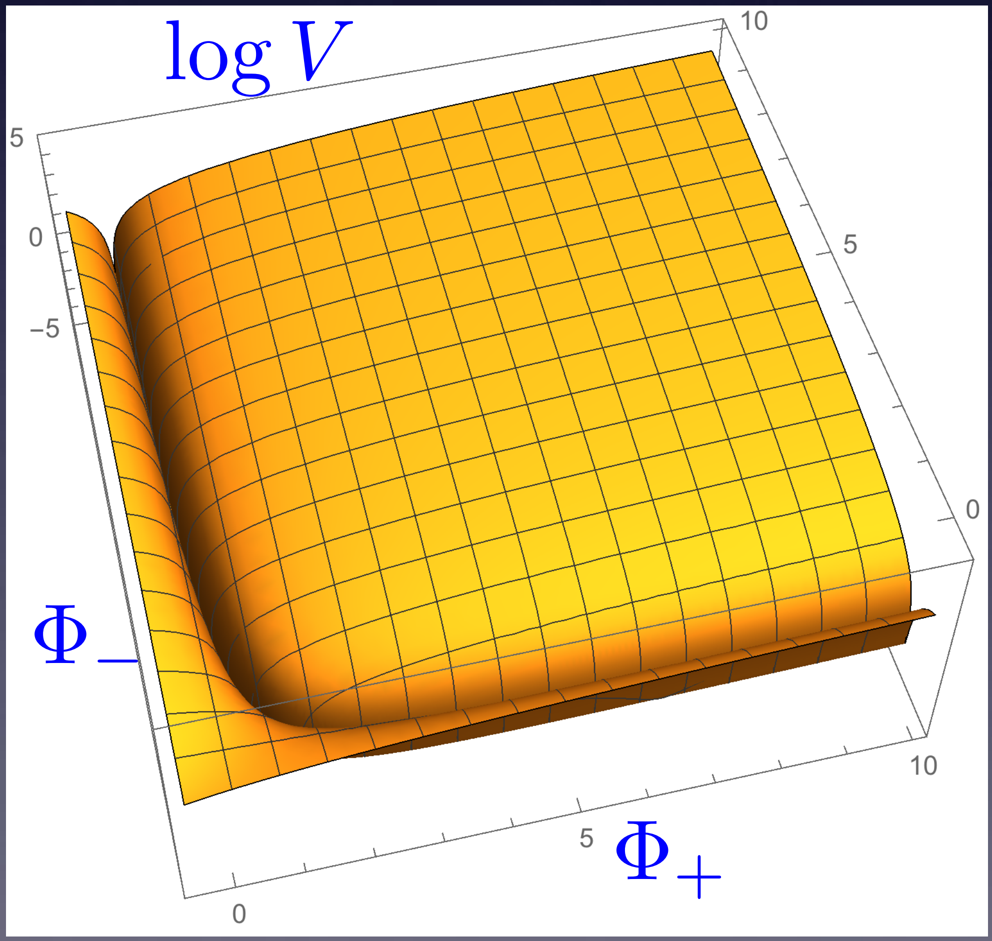
r < 0.02 for $N_{DW} = 6$

$H_{\rm inf} < 4 \times 10^{13}$ GeV



# 4.4 SUSY Axion Model

- Superpotential $$W = h\left(\Phi_+\Phi_- - \eta^2\right)\Phi_0$$
- PQ charges $\Phi_+(+1),\quad \Phi_-(-1),\quad \Phi_0(0)$
- Potential $$V = h^2\left|\Phi_+\Phi_- - \eta^2\right|^2 + |\Phi_0|^2(|\Phi_+|^2 + |\Phi_-|^2)$$ $$+ m_+^2|\Phi_+|^2 + m_-^2|\Phi_-|^2$$ $\Leftarrow$ soft SUSY breaking mass




- Flat direction $$\Phi_+\Phi_- = F_a^2, \quad \Phi_0 = 0$$
- Vacuum $$|\Phi_\pm| = \sqrt{\frac{m_\mp}{m_\pm}}\eta$$
- Axion

$$\Phi_\pm = \phi_\pm e^{i\theta_\pm}$$

$$a = \frac{\phi_+^2\theta_+ - \phi_-^2\theta_-}{\phi_+^2 + \phi_-^2}$$

$$F_a = \sqrt{\phi_+^2 + \phi_-^2}/N$$

# SUSY Axion Model

- During inflation PQ fields settle down to the flat direction
- If $\phi_+ \gg \eta \gg \phi_-$ → $a \simeq \phi_+ \theta_+ \quad \theta_a \simeq N\theta_+$
  - Isocurvature perturbations

$$\delta\theta_a \simeq \frac{N_{\mathrm{DW}}}{\phi_+}\left(\frac{H_{\mathrm{inf}}}{2\pi}\right)$$

→ Isocurvature perturbations are suppressed

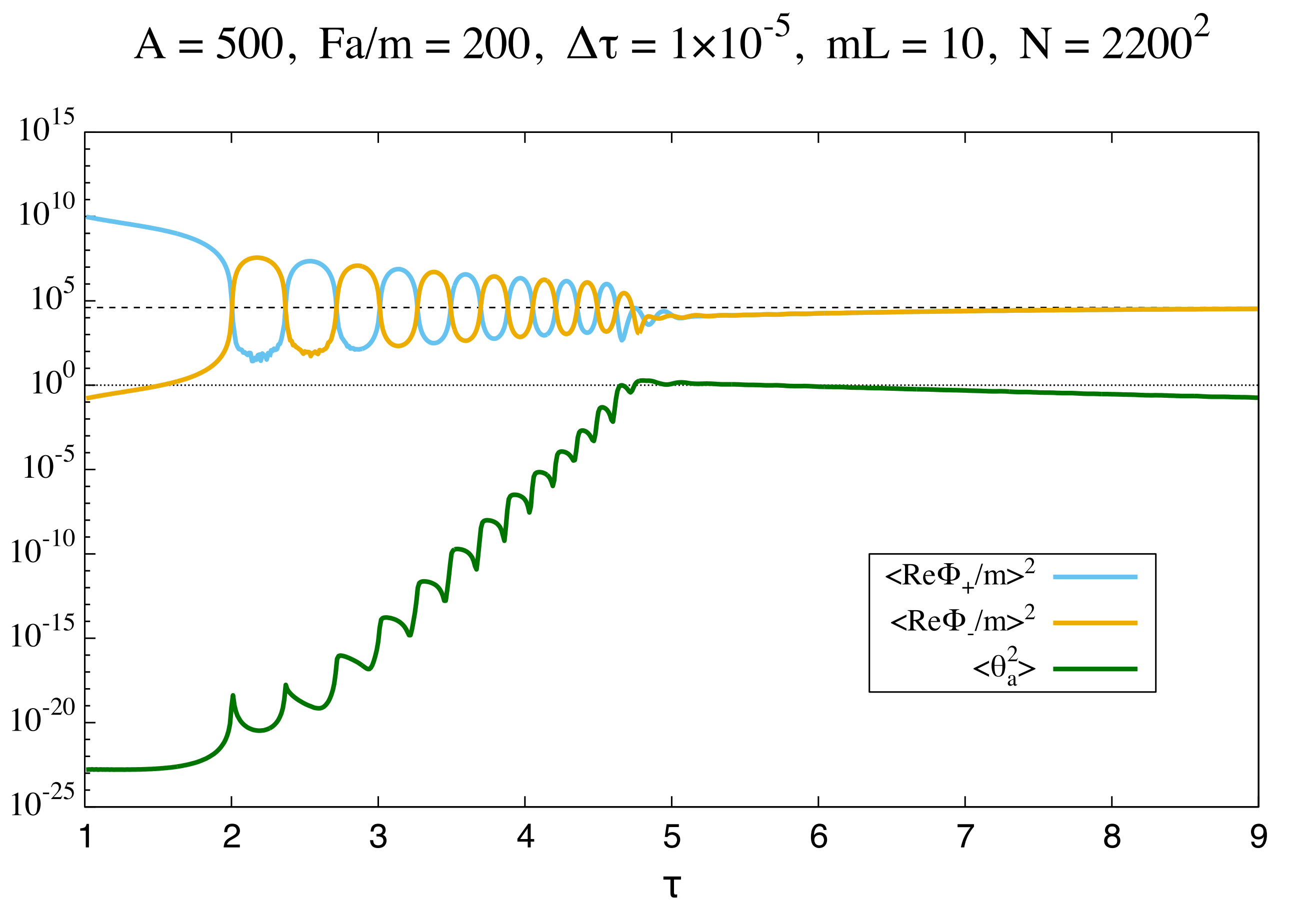
  - Parametric resonance takes place after inflation
  - Axion field can have large fluctuations
  - $U(1)_{PQ}$ is not restored because of the flat direction $\Phi_\pm \neq 0$

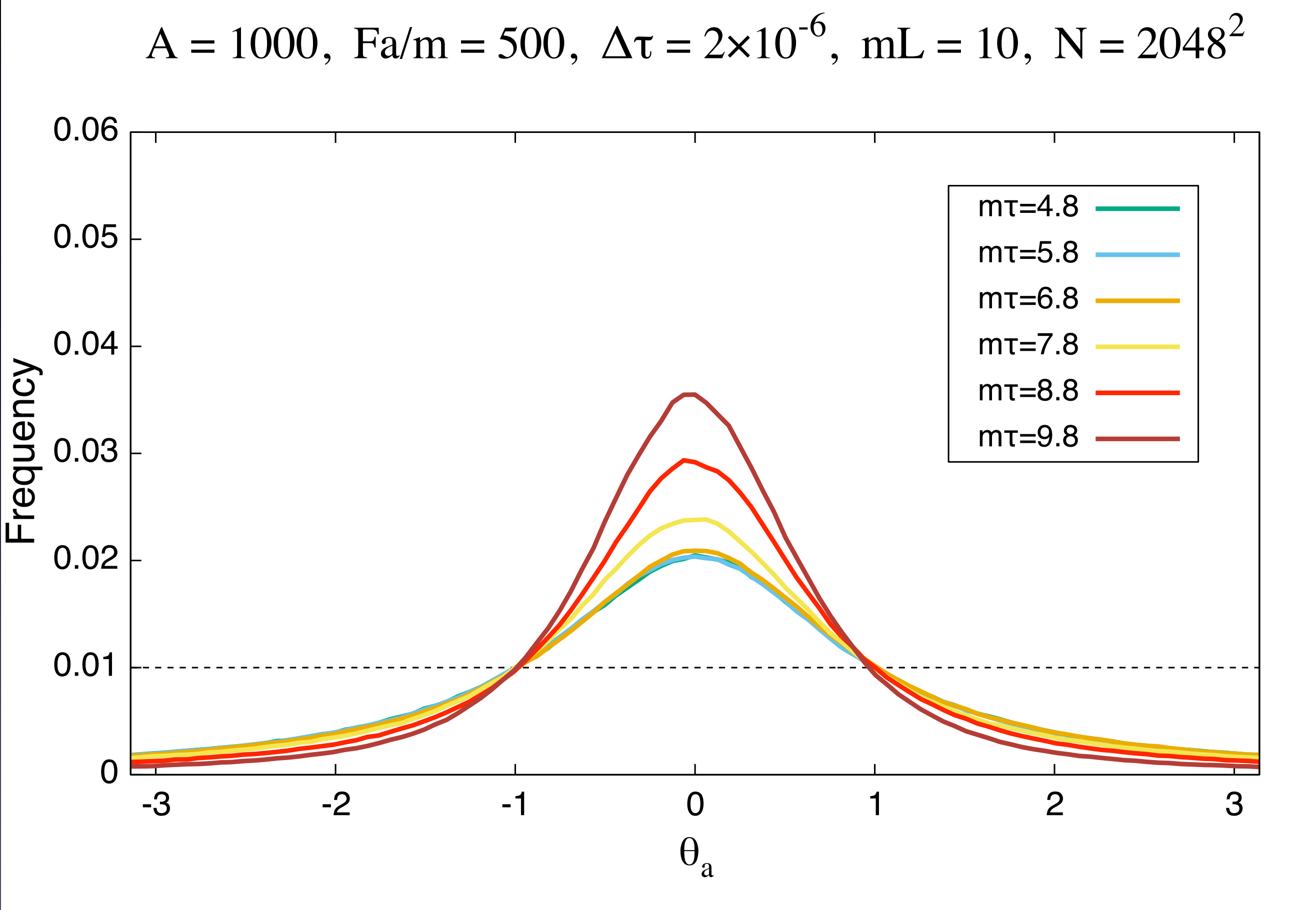
→ Domain walls without boundaries can be formed if misalignment angle is completely random

# SUSY Axion Model

- We have examined whether domain walls are formed using lattice simulations

MK, Sonomoto (2017)




- No domain walls are formed although we do not search the full parameter space

- Result suggests domain walls may not be formed

# 5. Conclusion

- If PQ symmetry is broken after inflation, topological defects are formed and axions from them give a significant contribution to the CDM density and axion can be dark matter for $F_a \sim 5X10^{10}$ GeV
- For domain wall number ≥ 2 there exist a serious domain wall problem which can be avoided by introducing a bias term and axon can be dark matter for lower PQ scales ( $F_a \sim 3X10^9 - 10^{10}$ GeV )
- If PQ symmetry breaks before or during inflation, axion has isocurvature density perturbations which are stringently constrained by CMB observations.
- If PQ scalar has a large field value during inflation, isocurvature perturbations can be suppressed and high scale inflation is consistent with axion dark matter