θ physics and axion

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θ term in QCD

$$S_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2} F^2 + \frac{i\theta}{32\pi^2} F\tilde{F} + \bar{\psi}(D+m)\psi \right)$$

$$Z_{\rm QCD}(\theta) = \int [dA][d\psi][d\bar{\psi}] e^{-S_{\rm QCD}}$$

instanton number: $Q = \int d^4x \frac{1}{32\pi^2} F\tilde{F}$ (integer!)

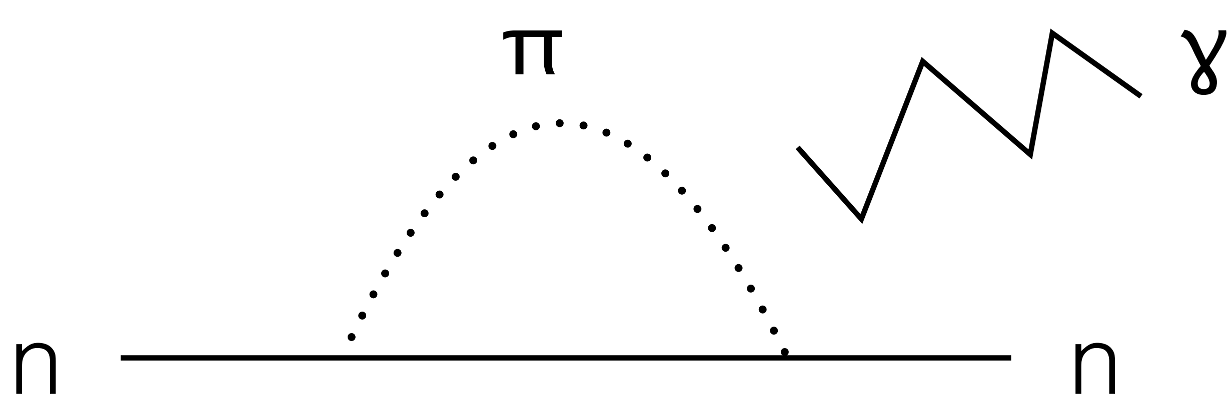
How does θ affect physics?

It is actually physical, and even known to be problematic.

Strong CP problem

θ term breaks CP

['t Hooft '76]



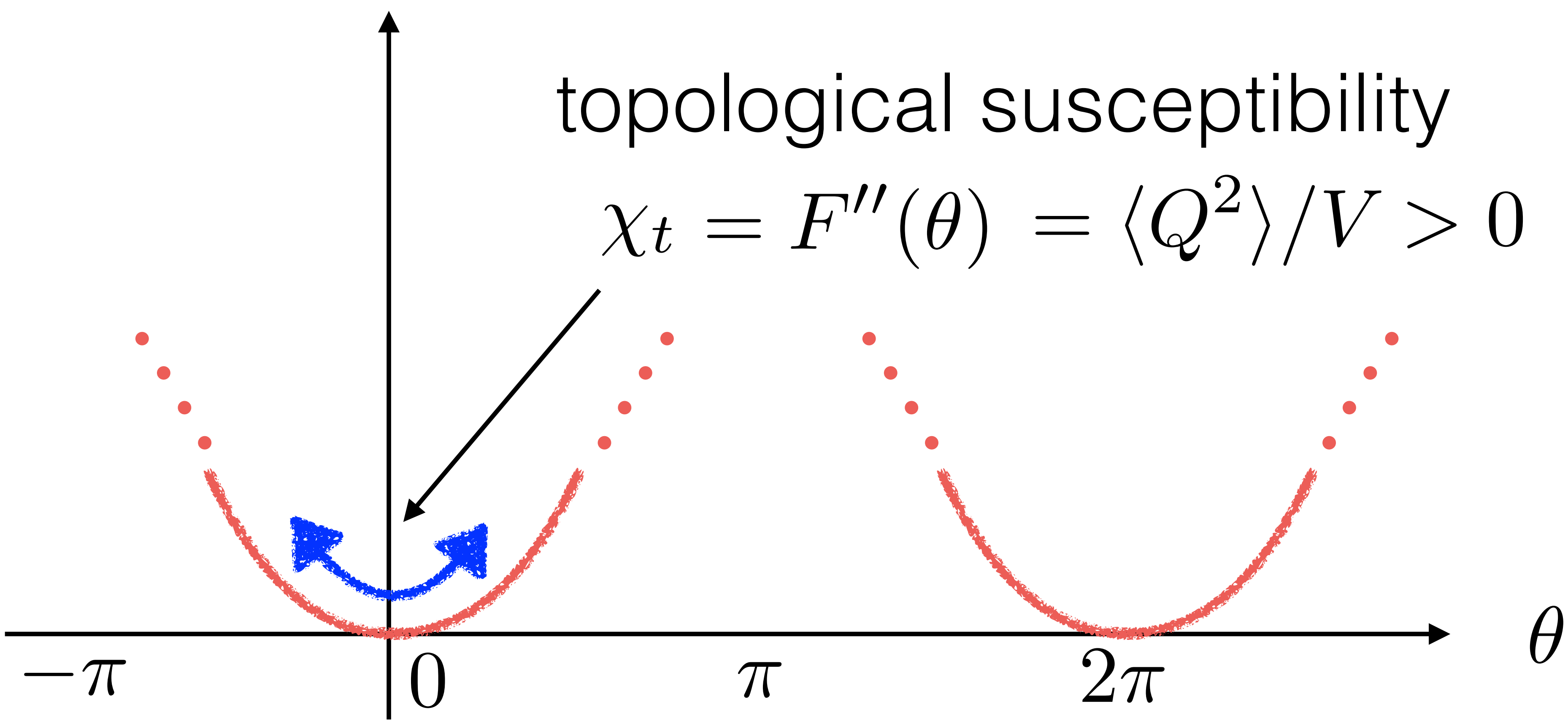
$$d_n \sim 10^{-15} \theta e \cdot \mathrm{cm}$$

$$\theta \lesssim 10^{-10}$$????

[Crewther, Di Vecchia, Veneziano, Witten '79]

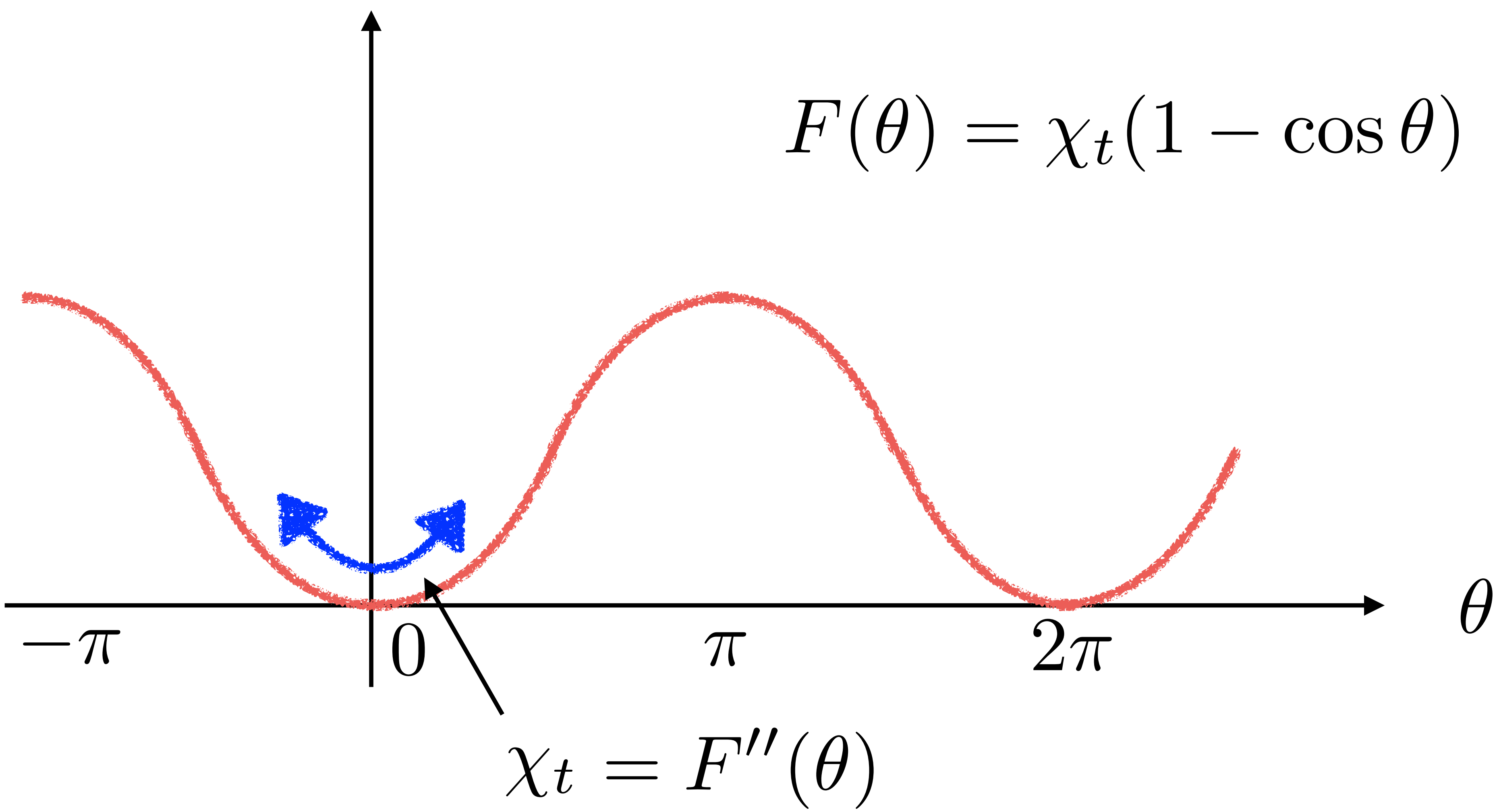
θ dependence of free energy?

$$F(\theta) = -\frac{1}{V}\log Z(\theta)$$



dilute instanton gas picture

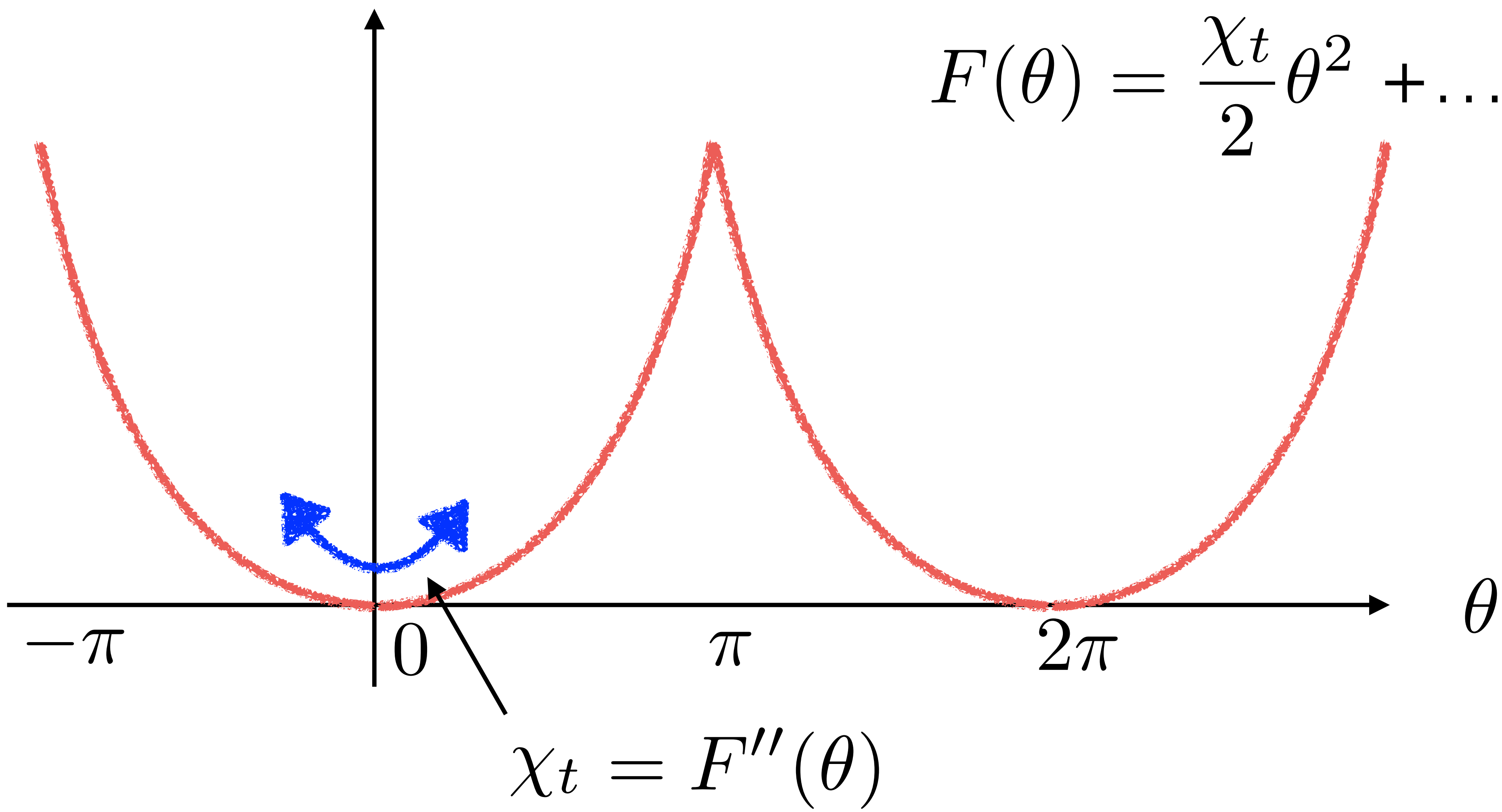
$$F(\theta) = -\frac{1}{V}\log Z(\theta)$$



[Witten ’98]

large N picture

$$F(\theta) = -\frac{1}{V}\log Z(\theta)$$



This seems to hold in YM theory with **finite** N. (maybe N>2)

[Gaiotto, Kapustin, Komargodski, Seiberg ’17] [RK, Suyama, Yamada ’17]

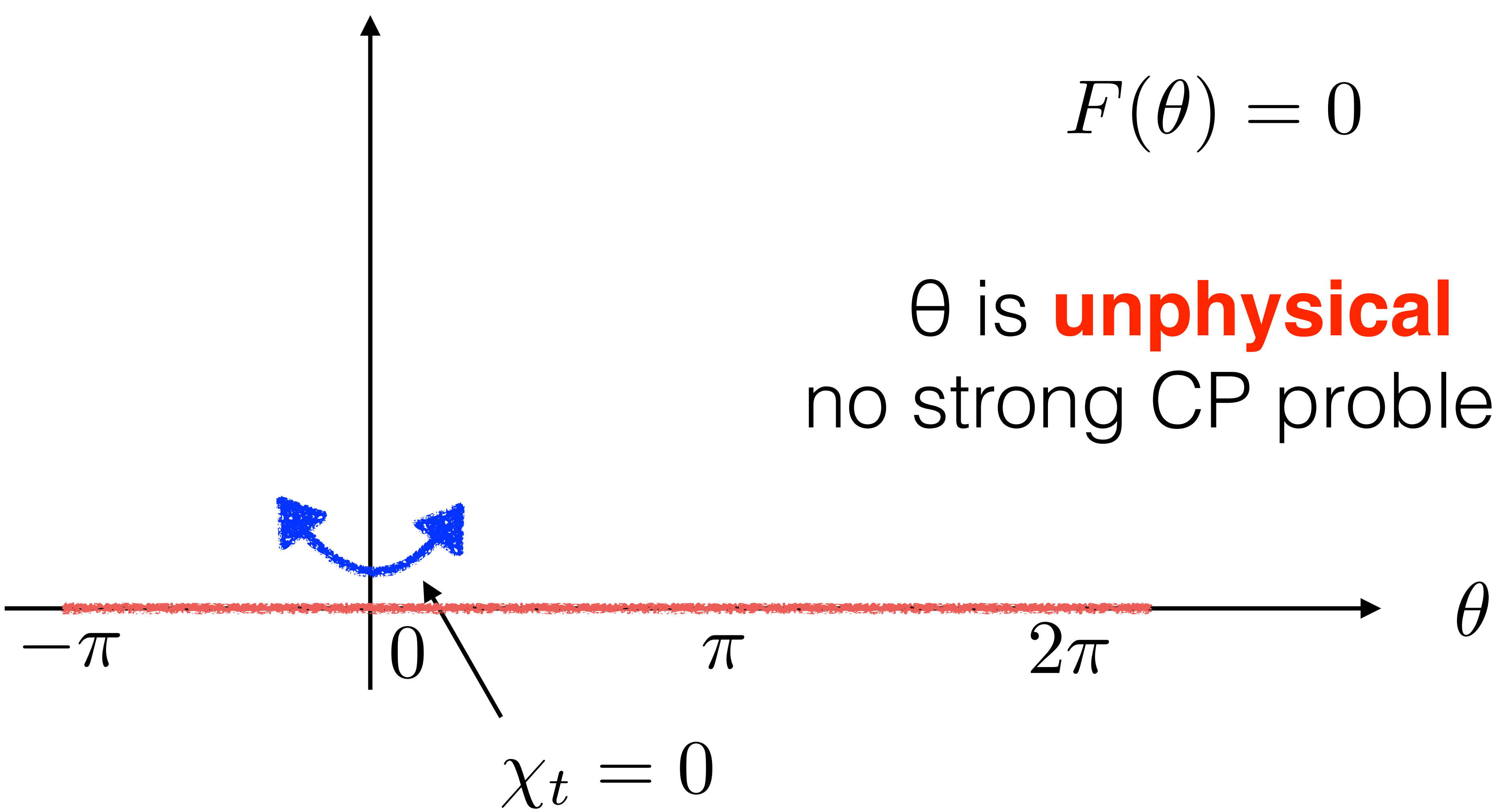
massless quark

$$F(\theta) = -\frac{1}{V}\log Z(\theta)$$

$$F(\theta) = 0$$

θ is **unphysical**

no strong CP problem



χ_t and quark mass

$$Z_{\rm QCD} = \int [dA][d\psi][d\bar{\psi}] e^{-S_{\rm QCD}} = \int [dA][d\psi][d\bar{\psi}] e^{-S'_{\rm QCD}}$$

$$S_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2} F^2 + \frac{i\theta}{32\pi^2} F\tilde{F} + \bar{\psi}(D+m)\psi \right)$$

$$S'_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2} F^2 + \bar{\psi}(D + m e^{-i\gamma_5\theta})\psi \right)$$

$$\chi_t = -\frac{1}{V}\frac{1}{Z}\frac{d^2 Z}{d\theta^2}\bigg|_{\theta=0} = -m_u \langle \bar{u}u \rangle + O(m_u^2/m_\pi^2)$$

If m_u is non zero, θ is physical.

If m_u=0, physics does **not** depend on θ.
—> no strong CP problem

m_u=0?

[PDG]

LIGHT QUARKS (*u*, *d*, *s*)

OMITTED FROM SUMMARY TABLE

u-QUARK MASS

The *u*-, *d*-, and *s*-quark masses are estimates of so-called "current-quark masses," in a mass- independent subtraction scheme such as $\overline{\text{MS}}$. The ratios m_u/m_d and m_s/m_d are extracted from pion and kaon masses using chiral symmetry. The estimates of *d* and *u* masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the *u* quark could be essentially massless. The *s*-quark mass is estimated from SU(3) splittings in hadron masses.

We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
2.3 $^{+0.7}_{-0.5}$ OUR EVALUATION	See the ideogram below.			
2.15±0.03±0.10	[1] DURR	11	LATT	$\overline{\text{MS}}$ scheme
2.24±0.10±0.34	[2] BLUM	10	LATT	$\overline{\text{MS}}$ scheme

Lattice QCD?

The quark mass is something you **define**
it's not just a parameter in the Lagrangian.

One should define the quark mass so that

$$\chi_t \propto m_u$$

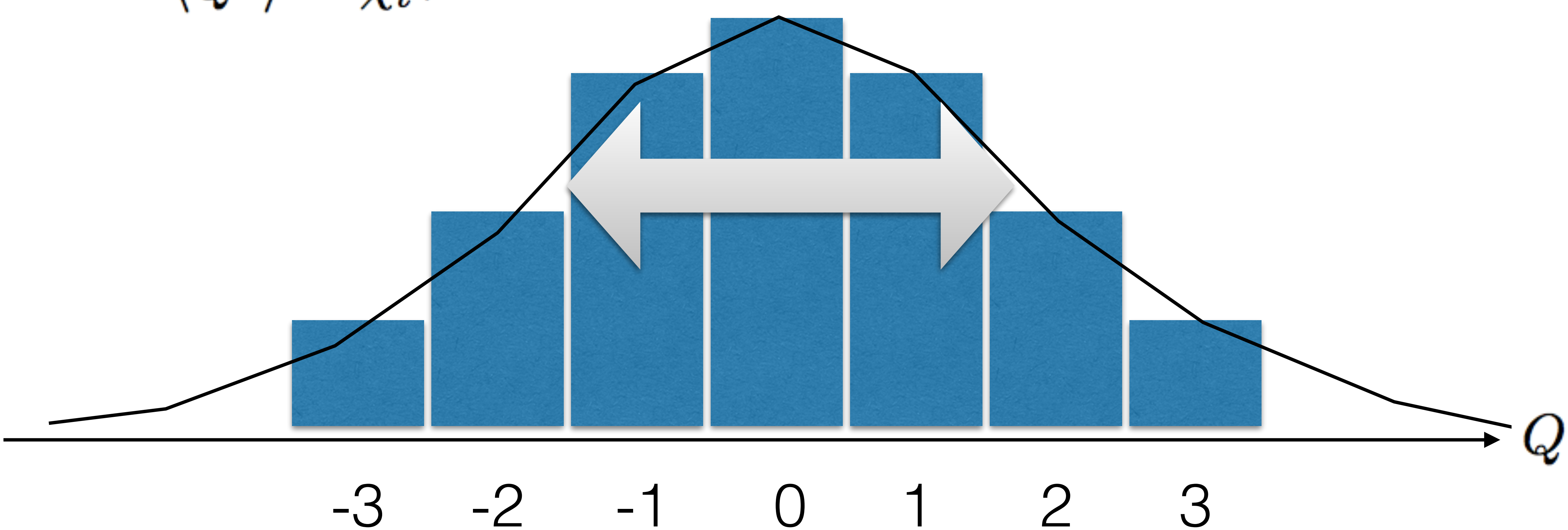
in order to establish

$m_u \neq 0$ and the strong CP problem is real.

χ_t on the lattice

$$Q = \int d^4x \frac{1}{32\pi^2} F\tilde{F} \quad = \mathrm{Tr}\gamma_5 = n_+ - n_-$$

$$\langle Q^2 \rangle = \chi_t V$$



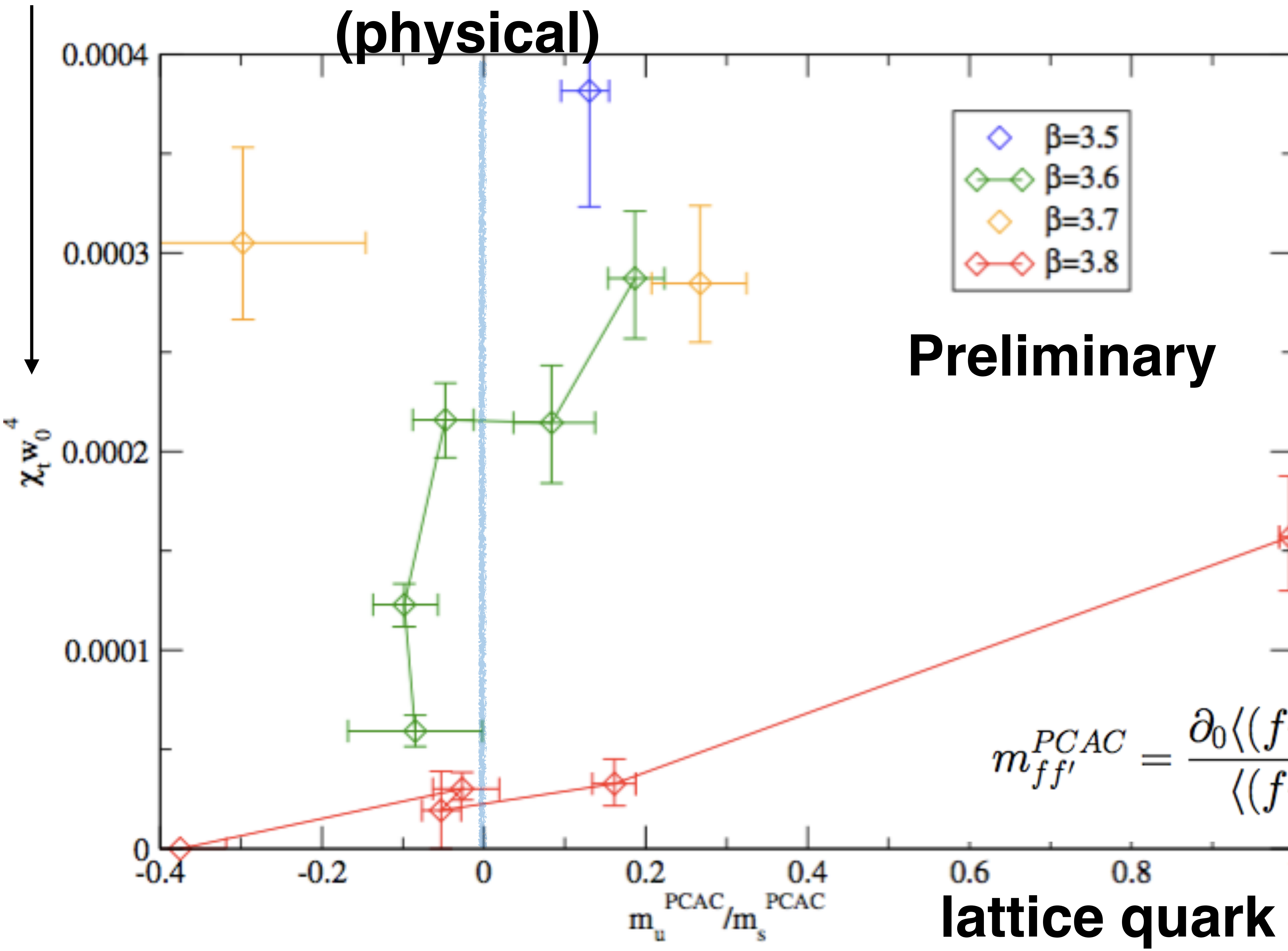
Let’s examine if the “lattice quark mass” satisfies

$$\chi_t \propto m_u$$

1+2 flavor QCD

[RK, Yamada, Frison '16]

measure of the Strong CP problem (physical)



Preliminary

$$m_{ff'}^{PCAC} = \frac{\partial_0 \langle (f\gamma_0\gamma_5 f')(f\gamma_5 f')\rangle}{\langle (f\gamma_5 f')(f\gamma_5 f')\rangle}.$$

lattice quark mass

It seems that there is a large discretization effects..
We need more data to say something quantitative.

Hopefully we can say something soon…

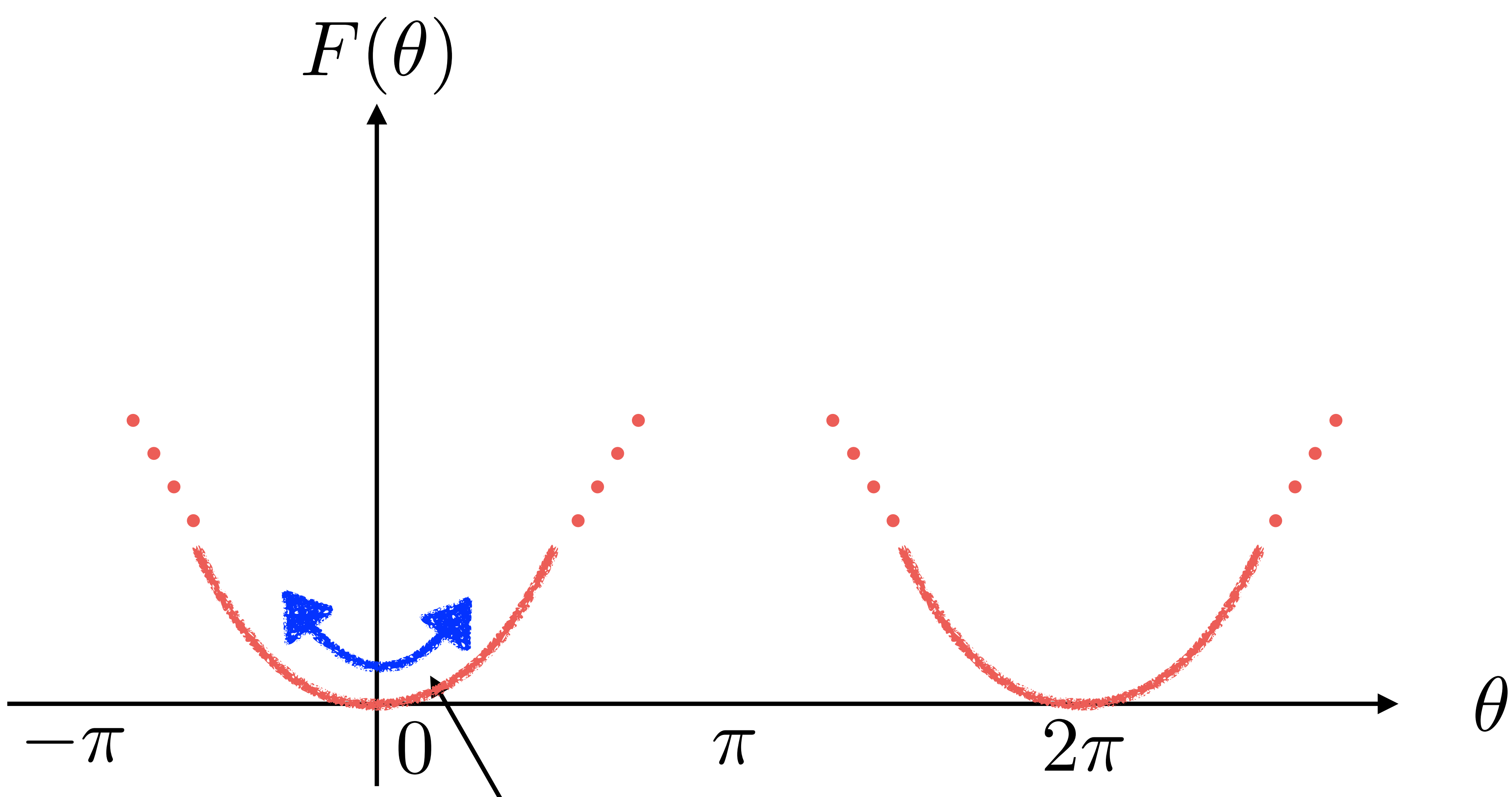
Sorry! No conclusion yet…

[Peccei, Quinn '77][Weinberg '78][Wilczek '78]

[Kim '79]
[Shifman, Vainstein, Zakharov '80]
[Zhitnitsky '80]
[Dine, Fischler, Srednicki '81]

Axion

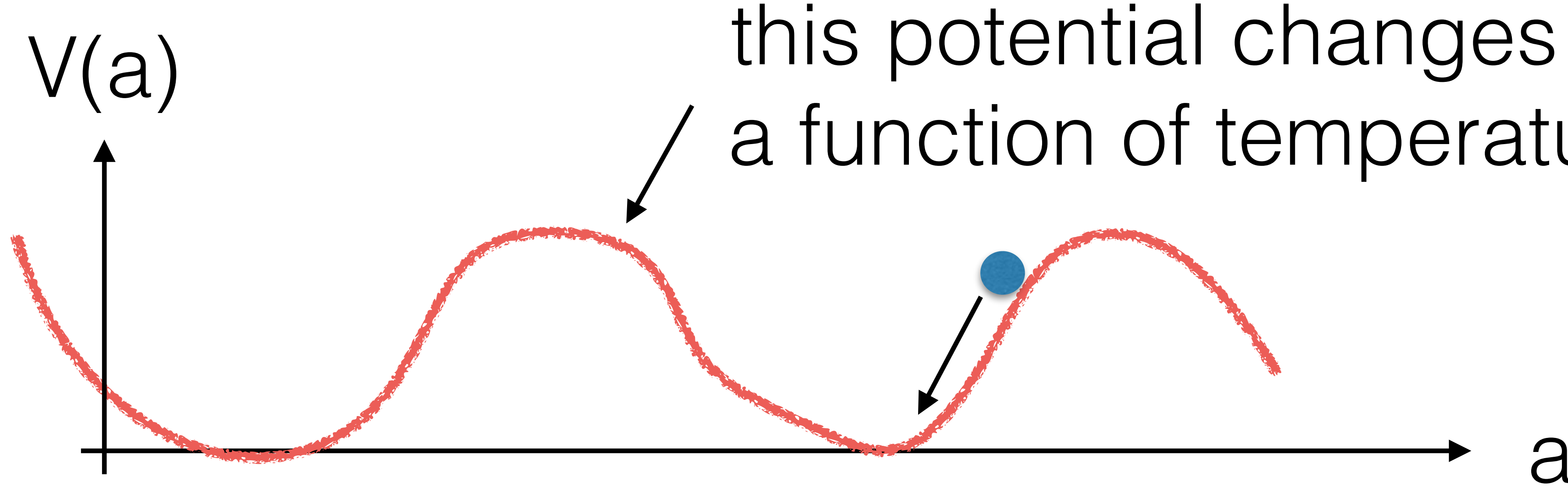
promote θ to a dynamical field → minimum of the potential preserve CP!



$\chi_t = F''(\theta) = m_a^2 f_a^2$ axion mass

[Preskill, Wise, Wilczek '83][Abbott, Sikivie '83]
[Dine, Fischler '83]

Axion Dark Matter



$$\ddot{a} + 3H\dot{a} = -V'(a) \sim -m_a^2 a$$

$$\Rightarrow \quad \left.\frac{n_a}{T^3}\right|_{\rm now} \sim \frac{m_a(T_*) f_a^2 \theta_{\rm ini}^2}{T_*^3}$$

where

$$m_a(T_*) \sim 3H(T_*)$$

temperature dependence of the axion mass
is the essential information to estimate the abundance.

instanton paradigm

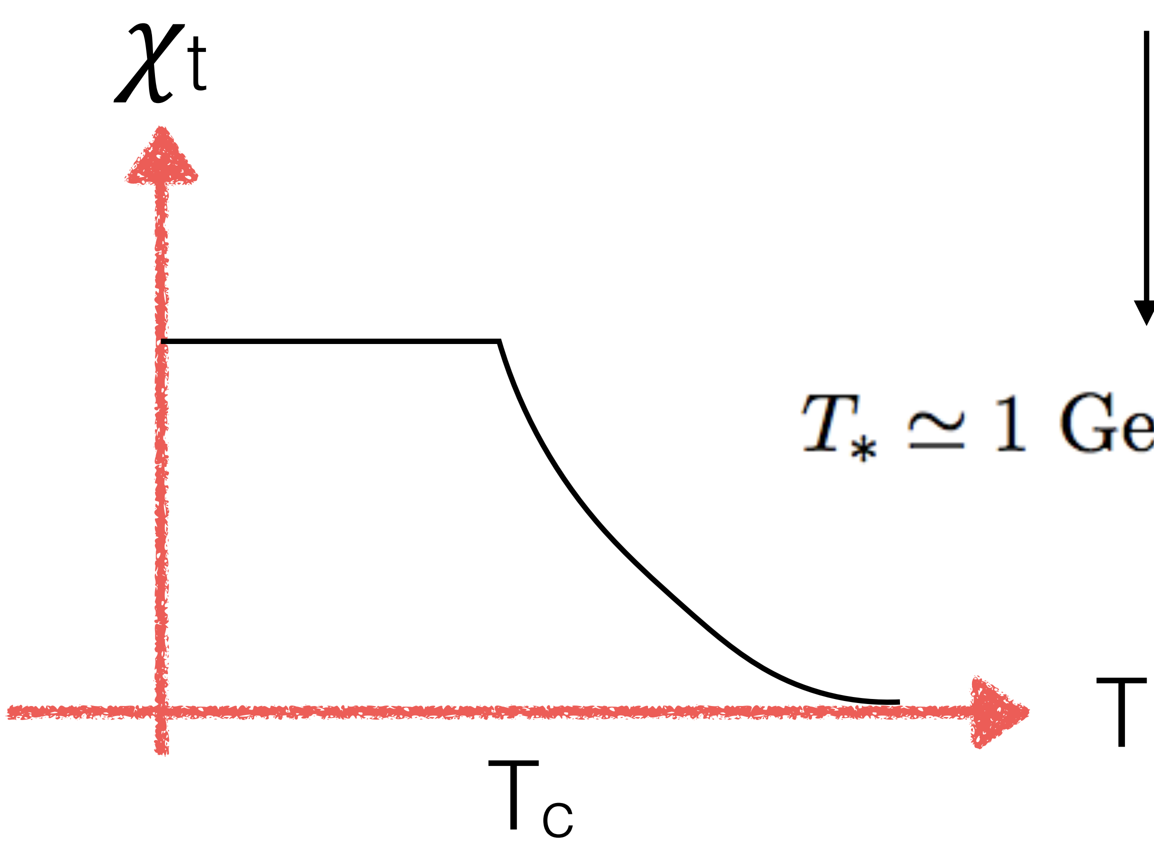
The standard way to calculate the temperature dependence of m_a is based on the dilute instanton gas approximation.

[Pisarsky, Yaffe '80]

$$\chi_t(T) = m_a^2(T) f_a^2 \propto m_u m_d m_s \Lambda_{\mathrm{QCD}}^b T^{-8}$$

$$b = \frac{11}{3}N_c - \frac{2}{3}N_f = 9$$

instanton action $e^{-8\pi^2/g^2}$

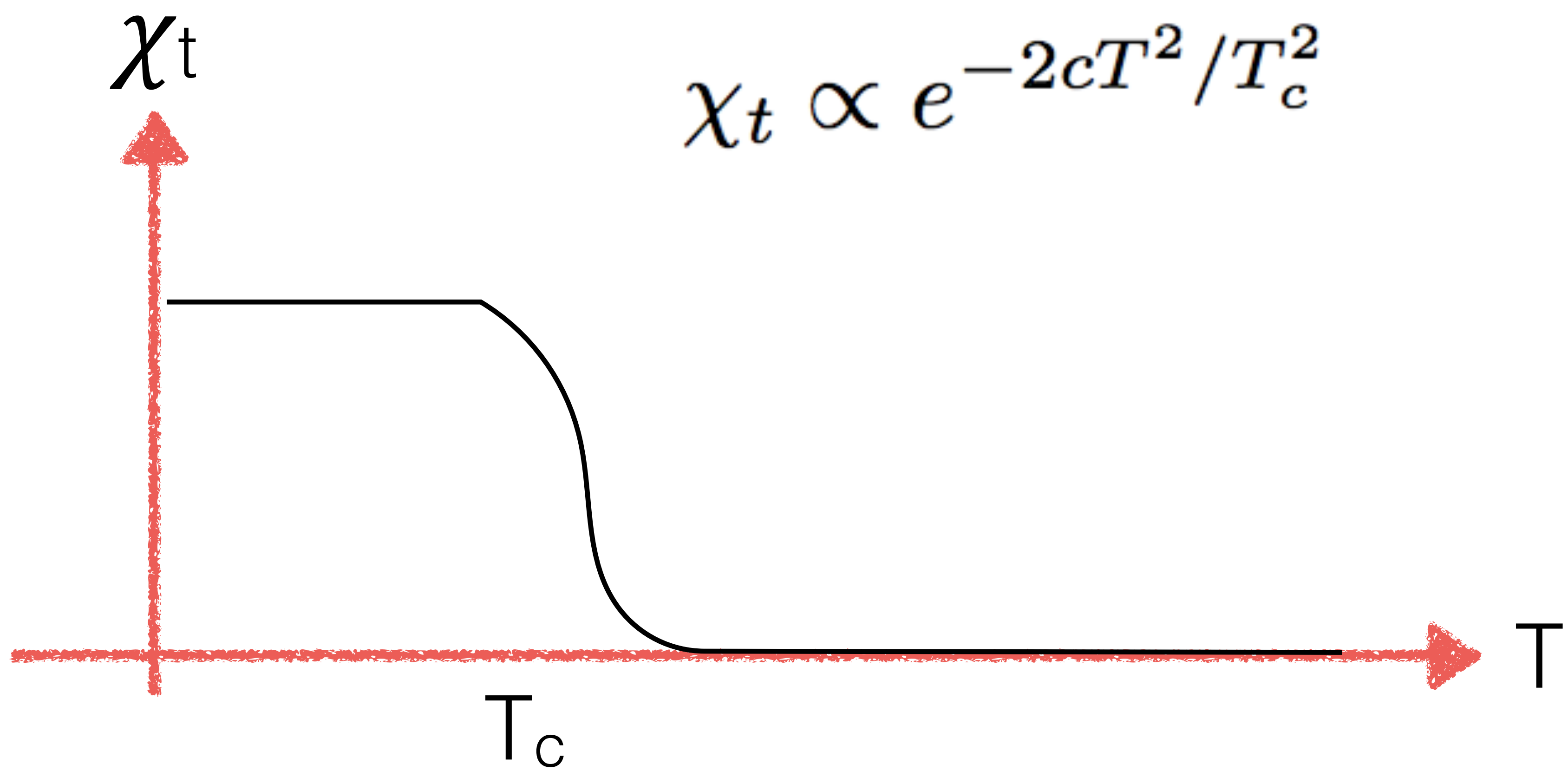


$$T_* \simeq 1\ \mathrm{GeV} \cdot \left(\frac{m_a}{10^{-5}\ \mathrm{eV}}\right)^{1/6}$$

$$\Omega_a \simeq 0.2 \cdot \theta_{\mathrm{ini}}^2 \left(\frac{m_a}{10^{-5}\ \mathrm{eV}}\right)^{-7/6}$$

if χ_t shuts off very quickly at T_c

the axion suddenly starts to oscillate at $T \sim T_c$

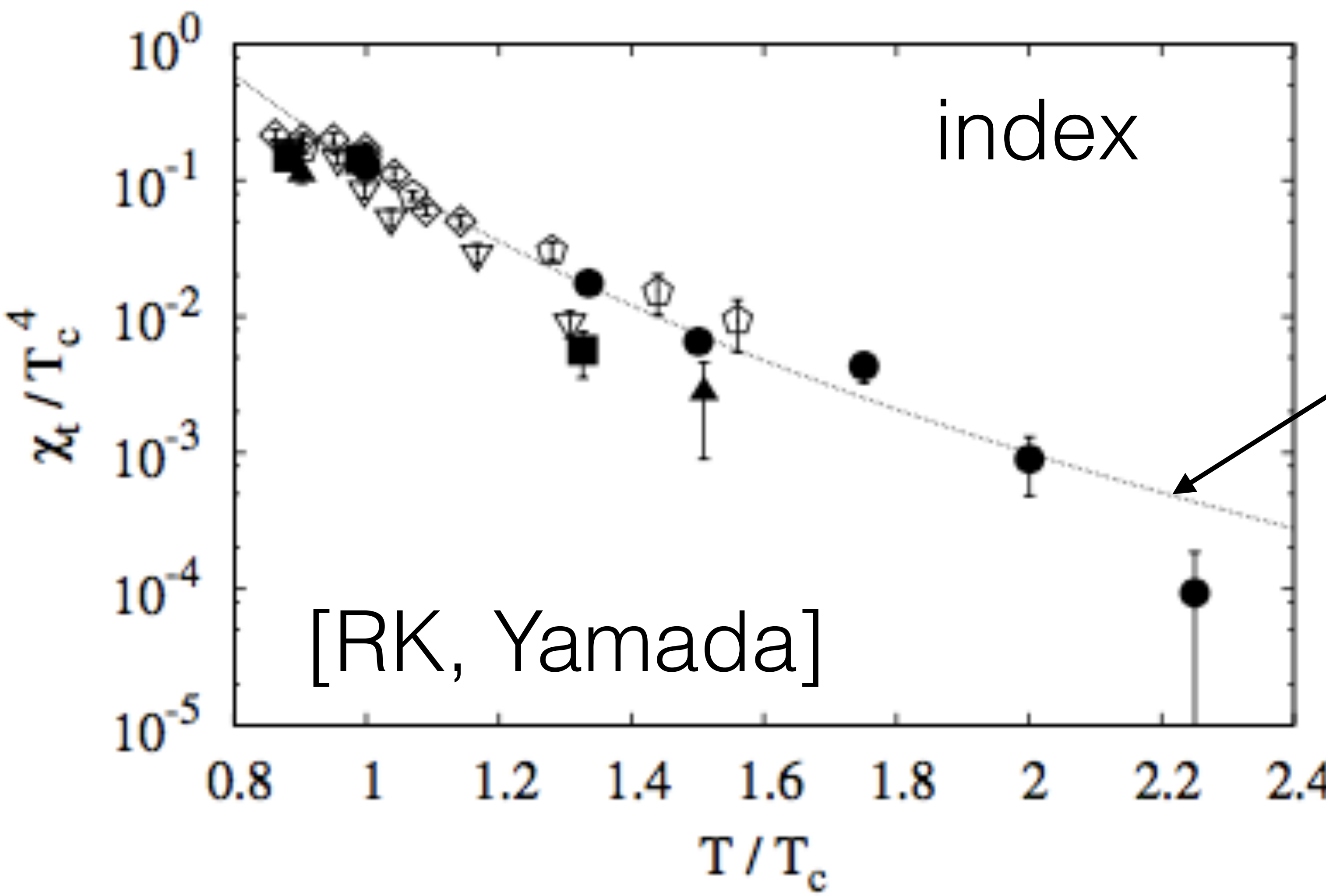


$$\Omega_a \sim 0.2\theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{-1} \times 2.5c \qquad (c \gg 1)$$

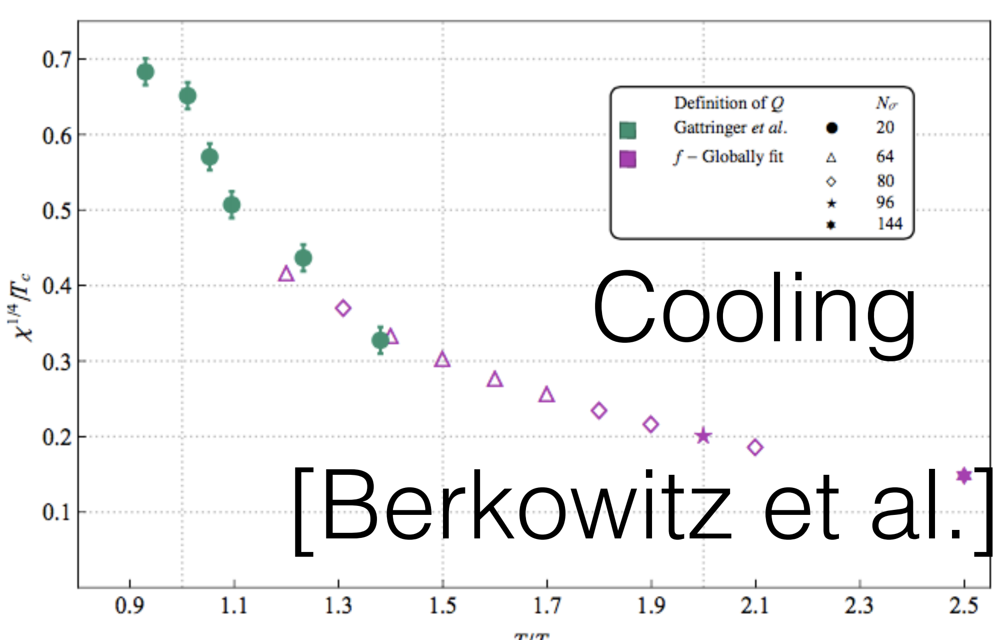
enhancement due to the **non-adiabatic evolution** of the potential.

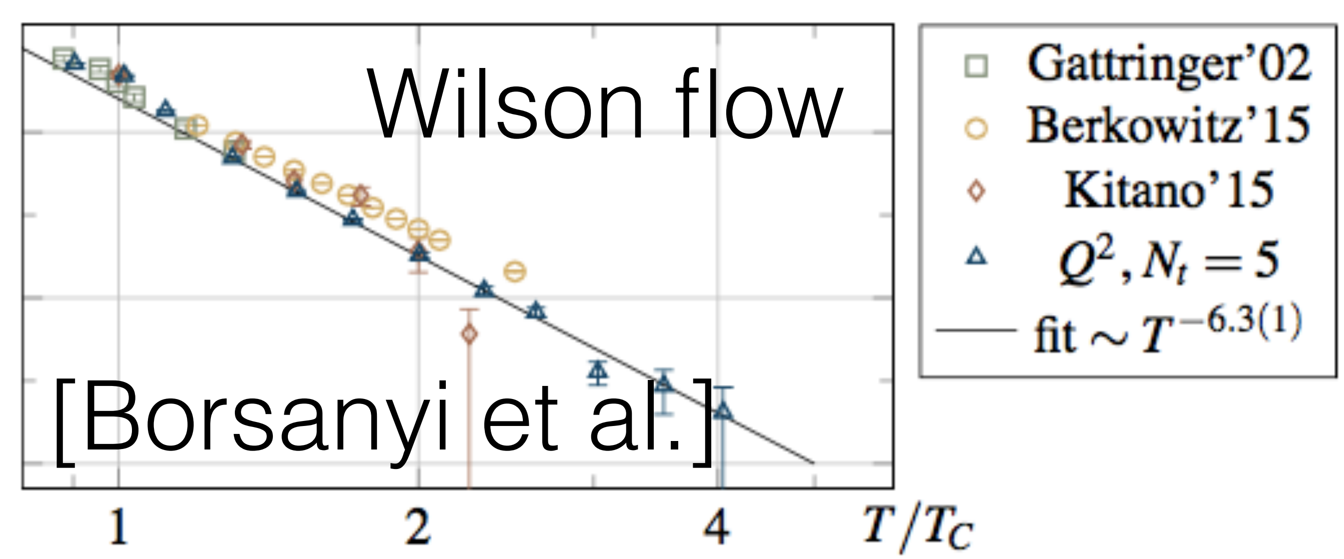
Dark Matter abundance is controlled by
How quickly the potential grows as temperature.

lattice results (SU(3) YM)



semi-classical
instanton estimates
(T^7)





Power Law

We see a clear **power law** even at a very low temperature.

The dilute Instanton gas picture seems to be pretty good above the critical temperature.

directly access the exponent

[Frison, RK, Matsufuru, Mori, Yamada '16]

$$\chi_t V(\beta) \simeq \frac{2Z_1(\beta)}{Z_0(\beta)} \qquad \chi_t(\beta) \propto T^k$$

$$\frac{d\ln Z_Q(T)}{d\ln T} = \left(\frac{d\beta}{d\ln T}\frac{\partial}{\partial\beta} + \frac{d\ln \bar{m}_q}{d\ln T}\frac{\partial}{\partial \ln \bar{m}_q}\right)\ln Z_Q(\beta, \bar{m}_q)$$

$$k = \frac{d\log\chi_t}{d\log T} = \frac{d\beta}{d\log T}(\langle S\rangle_{1,\beta} - \langle S\rangle_{0,\beta}) + 4$$

$$+N_f\left(1 + \frac{d\log m_q}{d\log a}\right) m_q\left(\langle\bar{q}q\rangle_{1,\beta} - \langle\bar{q}q\rangle_{0,\beta}\right)$$

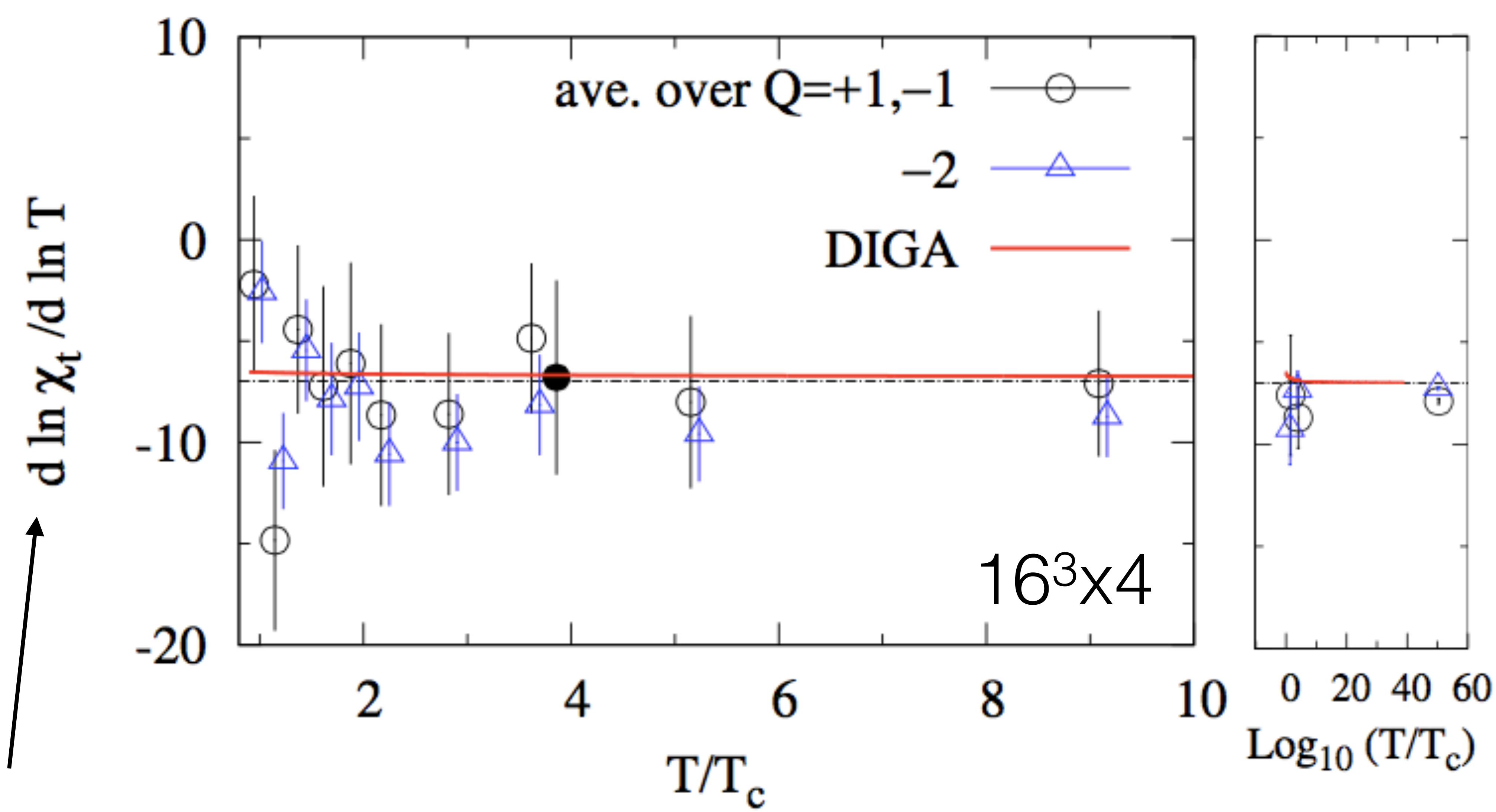
instanton prediction is "-b+4-Nf"

we can measure this by fixing the topology.

[Frison, RK, Matsufuru, Mori, Yamada '16]

Fixed topology measurements

(quenched)

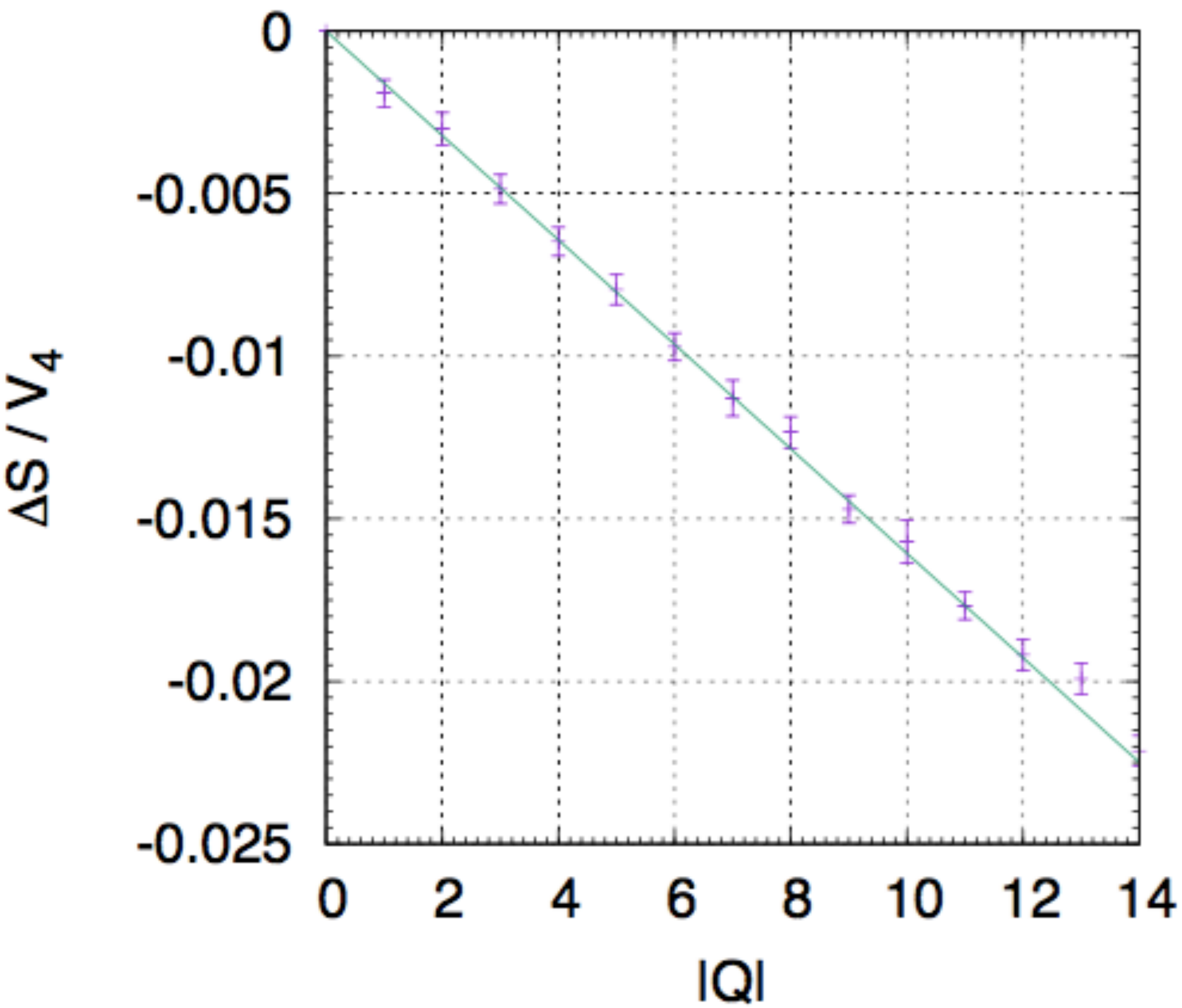


This quantity is related to the difference of <S> for fixed Q.

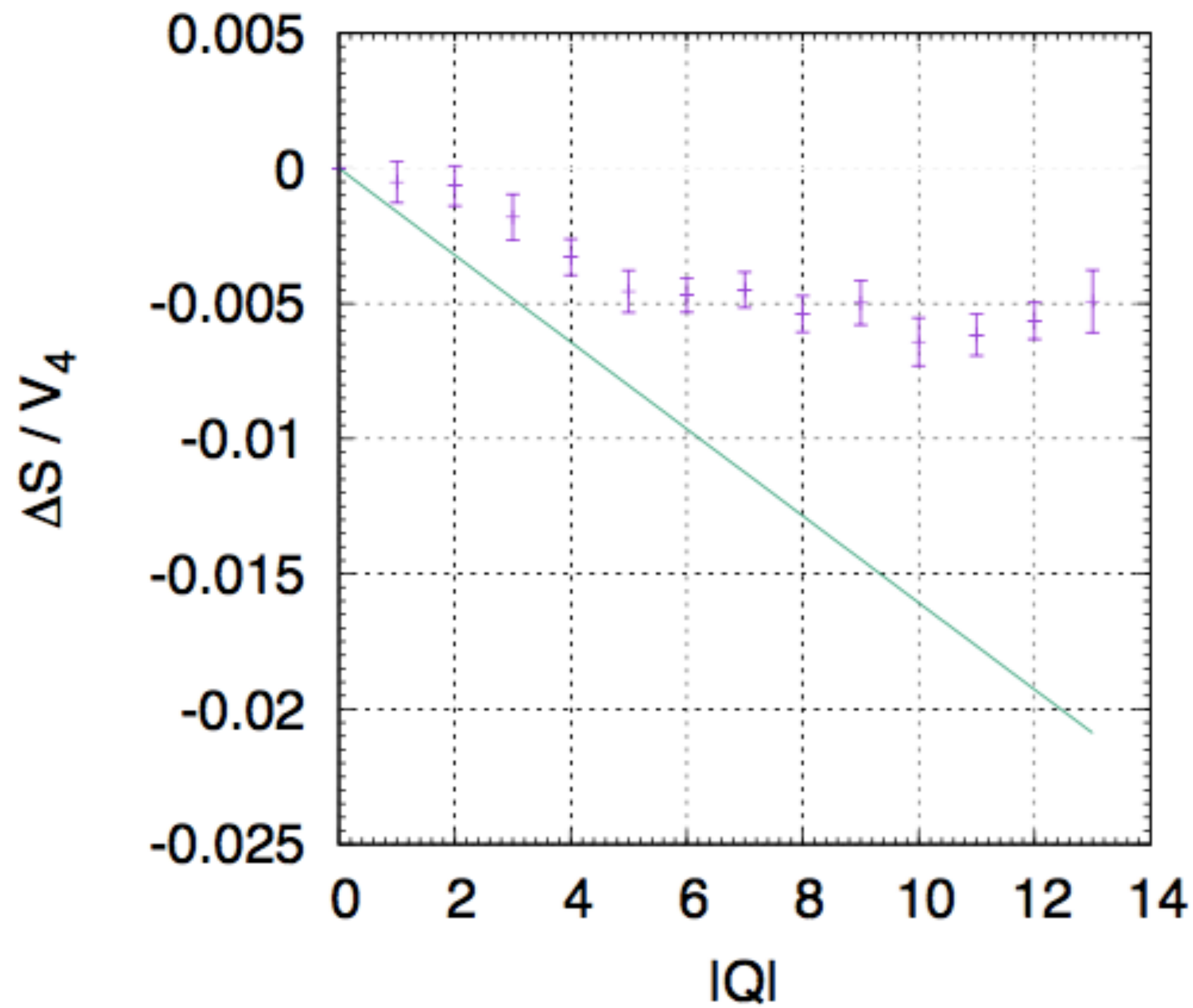
Instanton looks good.

fun with instantons

high temperature



low temperature

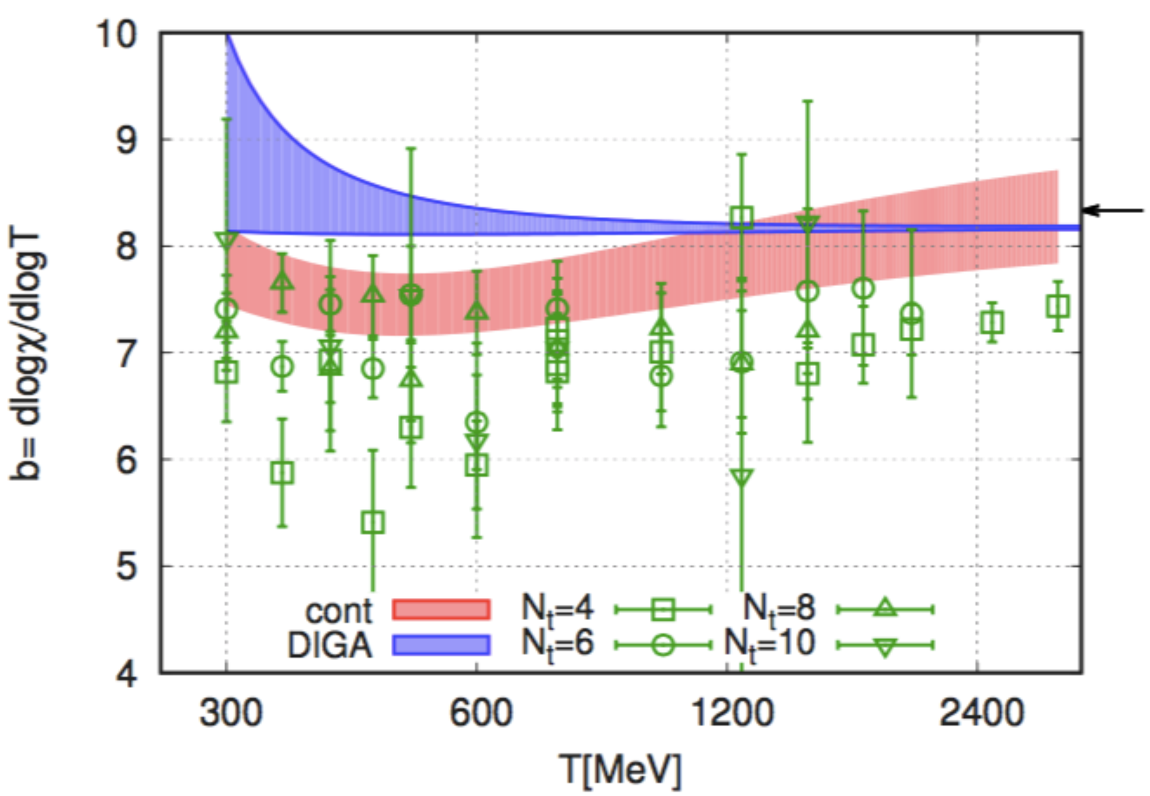


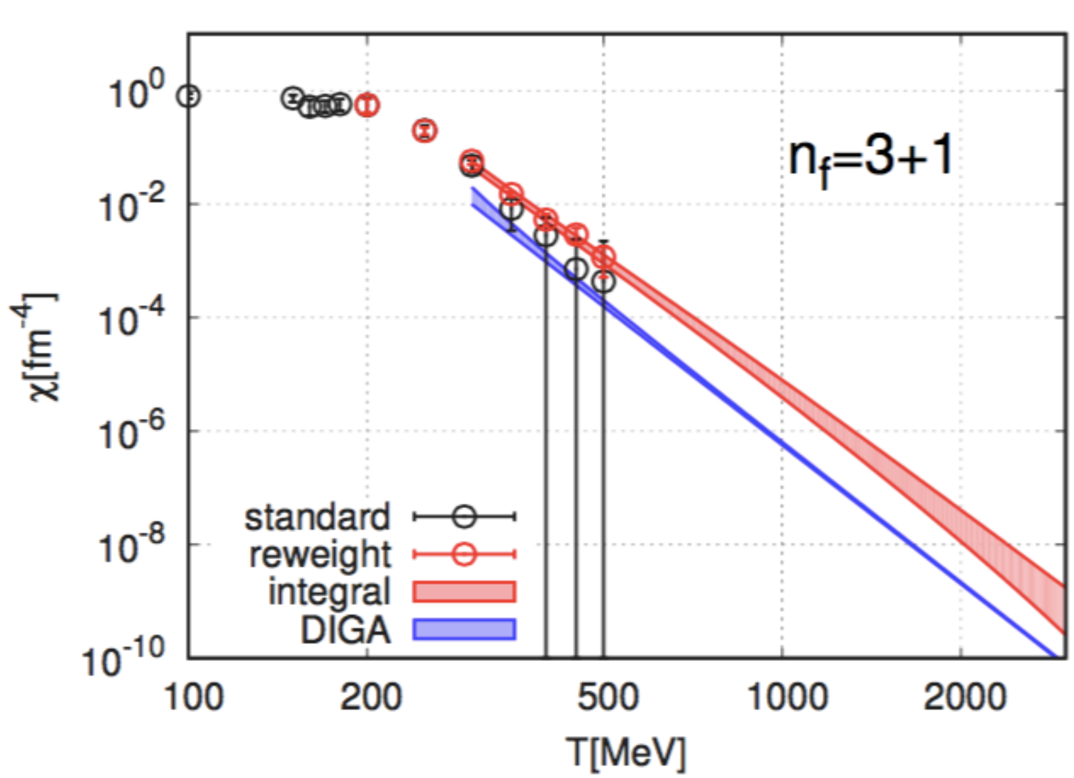
——— instanton gas predictions

dynamical fermions

from [1606.07494 Borsanyi et. al.]

$$\langle \overline{\psi}\psi_f \rangle^{\rm rw+zm}_{Q-0} = \langle \overline{\psi}\psi_f \rangle^{\rm rw}_{Q-0} + \frac{|Q|}{m_f} - \left\langle \frac{1}{2m_f} \sum_{n=1}^{2|Q|} \frac{4m_f^2}{\lambda_n^2[U] + 4m_f^2} \right\rangle^{\rm rw}_Q$$





Instanton picture looks essentially OK,
but normalization differs by O(10).

Instanton picture seems to work at high temperature.

(Actually, there are still active discussion on this point.)

See Fukaya's talk!

Summary

θ is physical

if quark is massive.

We have been considering

- Is quark really massive? (under progress..)
- How Axion mass behaves as a function of temperature? (looks like dilute instanton gas is good at least qualitatively)
- What's the actual θ dependence of the vacuum? Especially, what happens at $\theta=\pi$? (spontaneous CP violation or deconfinement!)