CAN AXIAL U(1) ANOMALY DISAPPEAR AT HIGH TEMPERATURE?

$$\langle \partial_\mu J_5^\mu \rangle = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \langle F^{\mu\nu} F^{\rho\sigma} \rangle \to 0?$$

HIDENORI FUKAYA (OSAKA UNIV.)

FOR JLQCD COLLABORATION

PRD96, NO.3, 034509(2017),

PRD93, NO.3, 034507 (2016)

DO YOU THINK AXIAL U(1) ANOMALY CAN DISAPPEAR (AT FINITE T) ?

YES	NO
$U(1)_A$ sym. may be at some T.	$U(1)_A$ is always broken.

DO YOU THINK AXIAL U(1) ANOMALY CAN DISAPPEAR (AT FINITE T) ?

Naïve answer would be "NO!"

with some reasonable reasons:

Anomaly = symmetry breaking at cut-off.

Anomalous Ward-Takahashi identity

$$\langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion} + \langle \delta_A O(x) \rangle_{fermion} \delta(x - x')$$

holds at any energy scale,

and for any gluon background.

DO YOU KNOW ANY OTHER ANOMALY WHICH CAN DISAPPEAR ?

YES	NO

DO YOU KNOW ANY OTHER ANOMALY WHICH CAN DISAPPEAR ?

The answer should be "YES".

Conformal anomaly in massless QCD

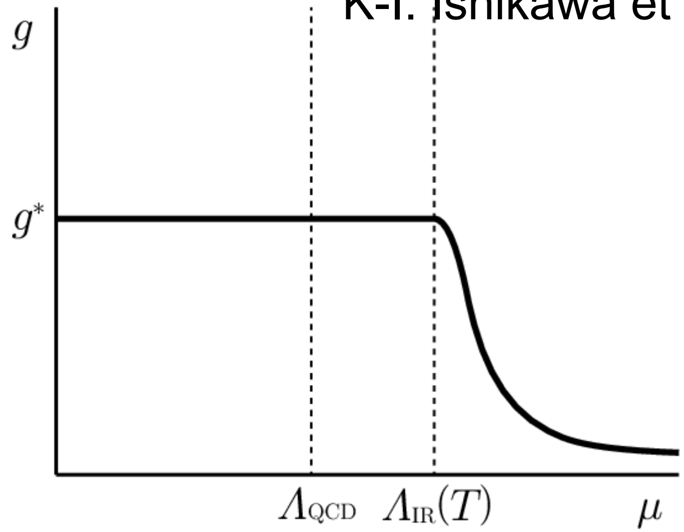
can disappear at IR

by tuning N_f

(conformal window).

Which Nf?→ nontrivial.

Figure from
K-I. Ishikawa et al. 2013



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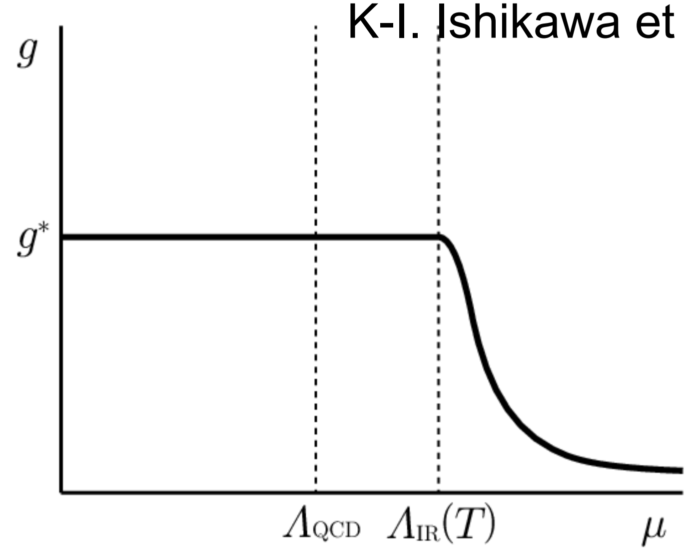

Why not axial U(1) (by tuning T ?)

WE ARE BIASED BY

$$\langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} - \langle \delta_A O(x) \rangle_{fermion} \delta(x - x')$$
$$= \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion}$$

BUT THE REAL QUESTION IS

$$\langle \langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} - \langle \delta_A O(x) \rangle_{fermion} \delta(x - x') \rangle_{gluons}$$

$$= \left\langle \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion} \right\rangle_{gluons} = 0???$$

MAIN MESSAGE OF THIS TALK

In high T QCD, whether

$$\left\langle \langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} - \langle \delta_A O(x) \rangle_{fermion} \delta(x - x') \right\rangle_{gluons}$$

$$= \left\langle \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion} \right\rangle_{gluons} = 0???$$

or not is a non-trivial question, which can only be answered by carefully integrating over gluons (by lattice QCD).

In particular, good control of chiral symmetry (or continuum limit) is essential.

CAN U(1)ANOMALY DISAPPEAR AT FINITE T? → MANY ANSWERS.

Before 2012

- Cohen 1996, 1998 (theory)
- Bernard et al. 1996 (staggered)
- Chandrasekharan et al. 1998 (staggered)
- HotQCD 2011 (staggered)
- Ohno et al. 2011 (staggered)
- and many others

Red: YES
Blue: NO
Green: Not (directly) answered but related

After 2012

- HotQCD 2012 (Domain-wall)
- Aoki-F-Taniguchi 2012 (theory)
- Ishikawa et al2013, 2014,2017. (Wilson)
- JLQCD 2013, 2016 (overlap)
- TWQCD 2013 (optimal DW)
- LLNL/RBC 2013 (Domain-wall) [may be at higher T]
- Pelisseto and Vicari 2013(theory)
- Bonati et al. 2014, 2016(staggered)
- Nakayama-Ohtsuki 2015, 2016(CFT)
- Sato-Yamada 2015(theory),
- Kanazawa & Yamamoto 2015, 2016 (theory)
- Dick et al. 2015 (OV in HISQ sea)
- Sharma et al. 2015, 2016 (OV in DW sea)
- Glozman 2015, 2016 (theory)
- Borasnyi et al. 2015 (staggered & OV)
- Brandt et al. 2016 (Wilson)
- Ejiri et al. 2016 (Wilson)
- Azcoiti 2016,2017(theory)
- Gomez-Nicola & Ruiz de Elvira 2017 (theory) ……

CONTENTS

$U(1)_A$ AND $SU(2)_L \times SU(2)_R$ SHARE DIM<=3 ORDER PARAMETER(S).

Among quark bi-linears $\langle \bar{q}\Gamma q(x) \rangle$

only $\langle \bar{q} q(x) \rangle$ can have a VEV :

No dim.<=3 operator breaks $U(1)_A$ without breaking $SU(2)_L x SU(2)_R$.

How about higher dim. operators ?

–> our work [Aoki, F, Taniguchi 2012]

DIRAC SPECTRUM AND SYMMETRIES [Aoki-F-Taniguchi 2012]

$$\langle \bar{q}q \rangle = \lim_{m \to 0} \int d\lambda \, \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi \rho(0)$$

[Banks-Casher 1980]

Our idea = generalization of BC relation

to higher dim operators (dim=6 operators were done by T.Cohen 1996) :

$$\delta_{SU(2)} \left\langle \frac{1}{V^{N'}} \prod_i^N \left(\int dV \bar{q} \Gamma_i q \right) \right\rangle = 0 \quad \Rightarrow \quad \text{Constraints on} \lim_{m \to 0} \rho(\lambda)$$

$$\Rightarrow \quad \delta_{U(1)_A} \left\langle \frac{1}{V^{N'}} \prod_i^N \left(\int dV \bar{q} \Gamma_i q \right) \right\rangle = 0 \; ???$$

[Aoki-F-Taniguchi 2012]

OUR RESULT 1 : MANY ORDER PARAMETERS ARE SHARED.

(under some “reasonable” assumptions)

Constraint we find

$$\lim_{m\to 0}\langle\rho(\lambda)\rangle = c|\lambda|^{\gamma}(1+O(\lambda)),\quad \gamma > 2$$

is strong enough to show

$$\delta_{U(1)_A}\left\langle \frac{1}{V^{N'}}\prod_i^N\left(\int dV\bar{q}\Gamma_i q\right)\right\rangle = 0 \text{ for } \Gamma_i{=}\tau^a \text{ and } \gamma_5\tau^a$$

for any N (up to 1/V corrections):

these order parameters are shared by $SU(2)_L \mathrm{x} SU(2)_R$ and $U(1)_A$.

[Aoki-F-Taniguchi 2012]

OUR RESULT 2 : STRONG SUPPRESSION OF TOPOLOGICAL SUSCEPTIBILITY

We also find (in the thermodynamical limit)

$$\left(\frac{\partial}{\partial m}\right)^N \frac{\langle Q^2 \rangle}{V} = 0 \quad \text{for any } N,$$

$$Q = \frac{1}{32\pi^2}\int d^4x \epsilon_{\mu\nu\rho\sigma} \mathrm{tr}[F^{\mu\nu}F^{\rho\sigma}]$$

which implies

$$\frac{\langle Q^2 \rangle}{V} = 0 \quad \text{for } m <^{\exists} m_{cr}$$

Suggests 1st order chiral transition ?

(There's no symmetry enhancement at finite quark mass.)

WHAT WE MEAN BY $U(1)_A$ "SYMMETRY"

We call it "symmetry" if

$$\langle \text{any } U(1)_A \text{ breaking} \rangle = \frac{1}{V^\alpha}, \quad \alpha > 0$$

Cf. conformal symmetry at the IR fixed point.

CONTENTS

✔ 1. **Is $U(1)_A$ "symmetry" theoretically possible ?**

   Our answer = YES. $SU(2)_L xSU(2)_R$ and $U(1)_A$ are connected through Dirac spectrum.

2. **Lattice QCD at high T with chiral fermions**

3. **Result 1: $U(1)_A$ anomaly**

4. **Result 2: topological susceptibility**

5. **Summary**

JLQCD COLLABORATION

Machines at KEK

HITACHI SR16000

Recently shut down …

IBM BG/Q

and U of Tsukuba

Oakforest-PACS

Simulation code = IroIro++

https://github.com/coppolachan/IroIro

JLQCD FINITE T PROJECT (2014-)

[JLQCD (Cossu et al.) 2015, JLQCD(Tomiya et al.) 2016]

We simulate 2-flavor lattice QCD.

1. **good chirality :**

   Mobius domain-wall & overlap fermion w/ OV/DW reweighting (frequent topology tunnelings)

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3. **different lattice spacings : 0.07-0.1 fm.**

Other comments

T= 190-330MeV (Tc~180MeV) with Lt=8,10,12.

3-10 different quark masses (w/ reweighting).

long MD time 20000-30000 for reweighting.

OVERLAP VS DOMAIN-WALL

Overlap Dirac operator has exact chiral symmetry

$$D_{\mathrm{ov}}(m) = \left[\frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \,\mathrm{sgn}(H_M)\right]$$

Measure for how much chiral sym. is violated

$$m_{\mathrm{res}} = 0.$$

(Monius) domain-wall operator is an approximation of overlap.

$$D^{4D}_{\mathrm{DW}}(m) = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \frac{1-(T(H_M))^{L_s}}{1+(T(H_M))^{L_s}}$$

$$m_{\mathrm{res}} \sim 1\mathrm{MeV}$$

$$H_M = \gamma_5 \frac{2D_W}{2+D_W}$$

We thought domain-wall fermion was good enough. But...

VIOLATION OF CHIRAL SYMMETRY ENHANCED AT FINITE TEMPERATURE

[JLQCD (Cossu et al.) 2015, JLQCD(Tomiya et al.) 2016]

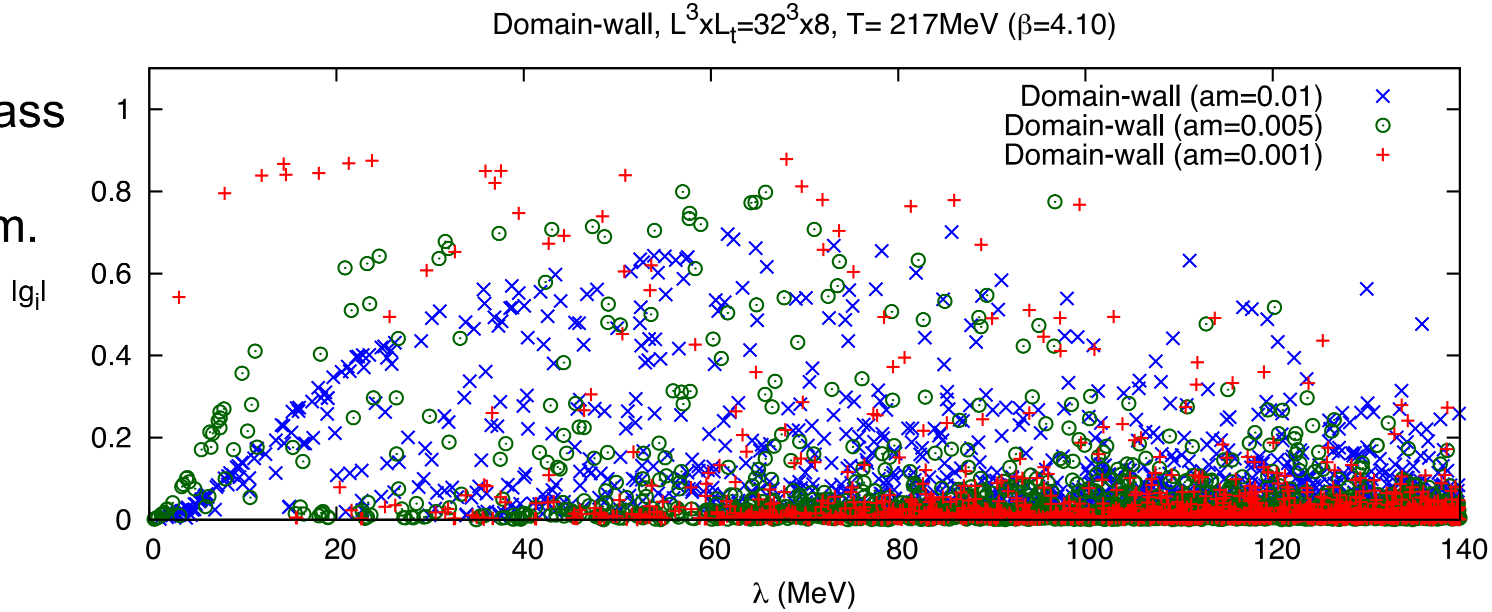
Examine chiral symmetry for each eigen-mode of Mobius domain-wall Dirac operator:

$$g_i = \left(v_i^\dagger, \frac{D\gamma_5 + \gamma_5 D - aRD\gamma_5 D}{\lambda_i} v_i \right)$$

$\rightarrow$ **very bad modes appear above Tc (∼180MeV).**

Cf.) residual mass is (weighted) average of them.

For T=0, gi are consistent with residual mass.



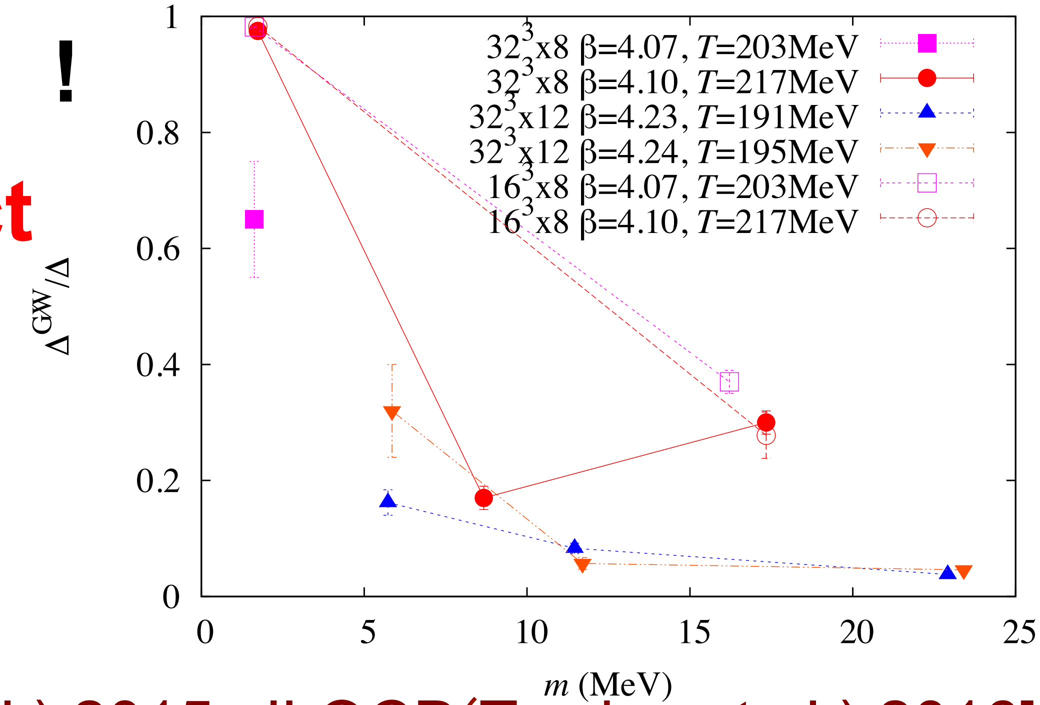
$U(1)_A$ ANOMALY IS SENSITIVE TO THE BAD MODES.

Mobius domain-wall fermion is not good enough (at high T) !

GW violation effect is 20%-100% .

(10 times of m_{res})

GW violation part in U(1)A susceptibility (definition will be given later.)

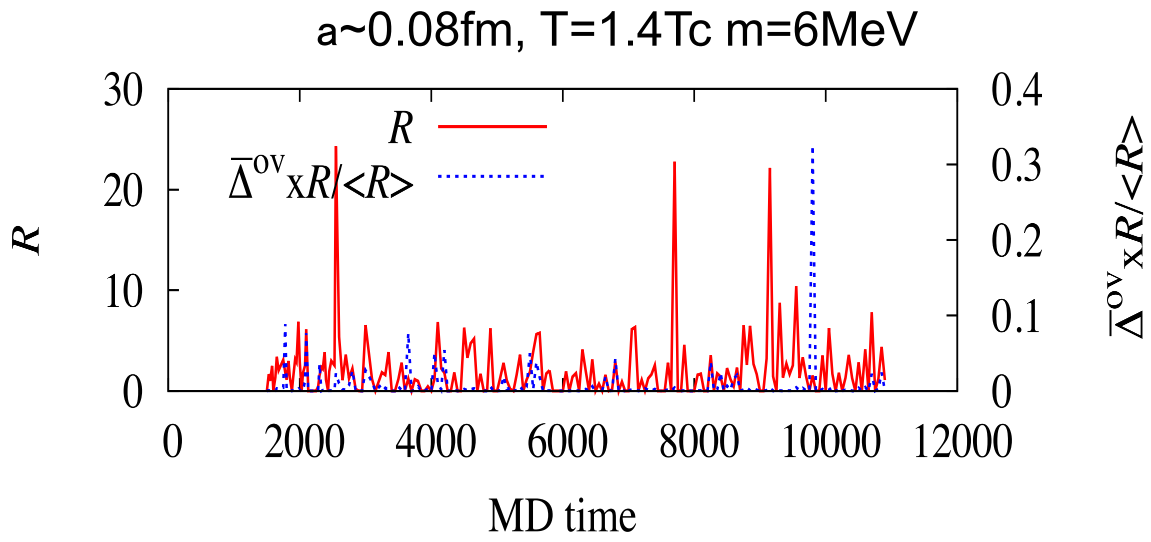


[JLQCD (Cossu et al.) 2015, JLQCD(Tomiya et al.) 2016]

OVERLAP/DOMAIN-WALL REWEIGHTING

(fermion action can be changed AFTER simulations)

$$R \equiv \frac{\det[D_{\rm ov}(m)]^2}{\det[D_{\rm DW}^{\rm 4D}(m)]^2}$$

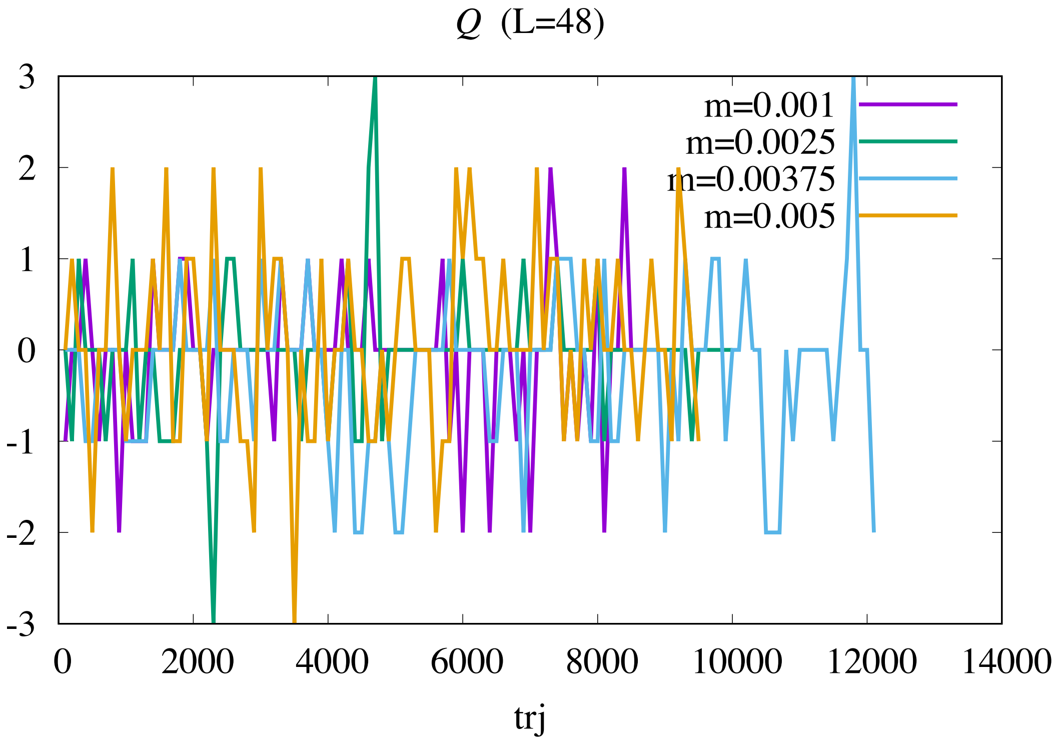


$$\langle O \rangle_{overlap} = \frac{\int dAO[\det D_{\rm ov}(m)]^2 e^{-S_G}}{\int dA[\det D_{\rm ov}(m)]^2 e^{-S_G}}$$

$$= \frac{\int dAOR[\det D_{\rm DW}^{\rm 4D}(m)]^2 e^{-S_G}}{\int dAR[\det D_{\rm DW}^{\rm 4D}(m)]^2 e^{-S_G}}$$

$$= \frac{\langle OR \rangle_{domain-wall}}{\langle R \rangle_{domain-wall}}$$

OVERLAP/DOMAIN-WALL REWEIGHTING ALLOWS TOPOLOGY TUNNELINGS



L=48 (3.6fm)
a = 0.07fm
T = 220 MeV

Auto-correlation time of topology is O(100), small enough compared to our long trajectory length, 20000-30000 MD time.

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Our answer = YES. $SU(2)_L xSU(2)_R$ and $U(1)_A$ are connected through Dirac spectrum.

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$U(1)_A$ at high T is sensitive to lattice artifact.

We need good chiral sym (or careful cont. limit.).

3. **Result 1: $U(1)_A$ anomaly**

4. **Result 2: topological susceptibility**

5. **Summary**

WHAT WE OBSERVE

Axial U(1) susceptibility

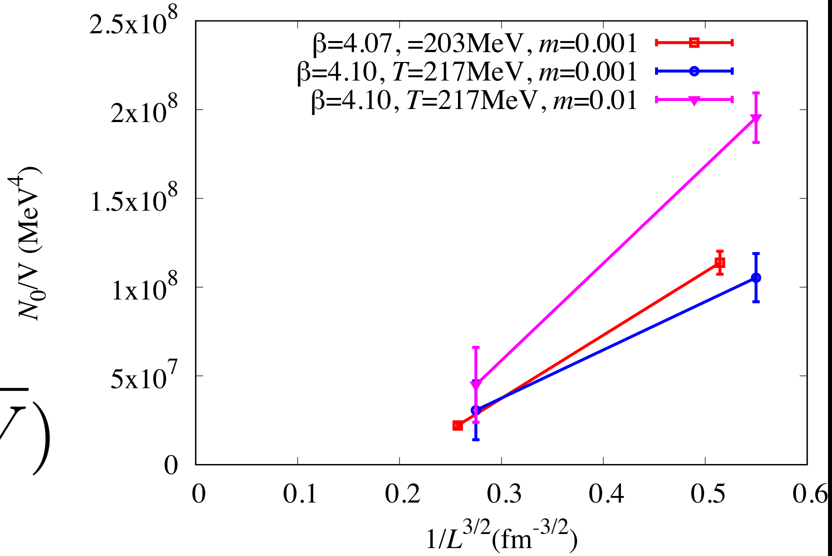
$$\Delta_{\pi-\delta} = \int d^4x \, [\langle \pi^a(x)\pi^a(0)\rangle - \langle \delta^a(x)\delta^a(0)\rangle] \, ,$$

$$\left(= \int_0^\infty d\lambda \, \rho(\lambda) \frac{2m^2}{(\lambda^2 + m^2)^2} \right)$$

We compute

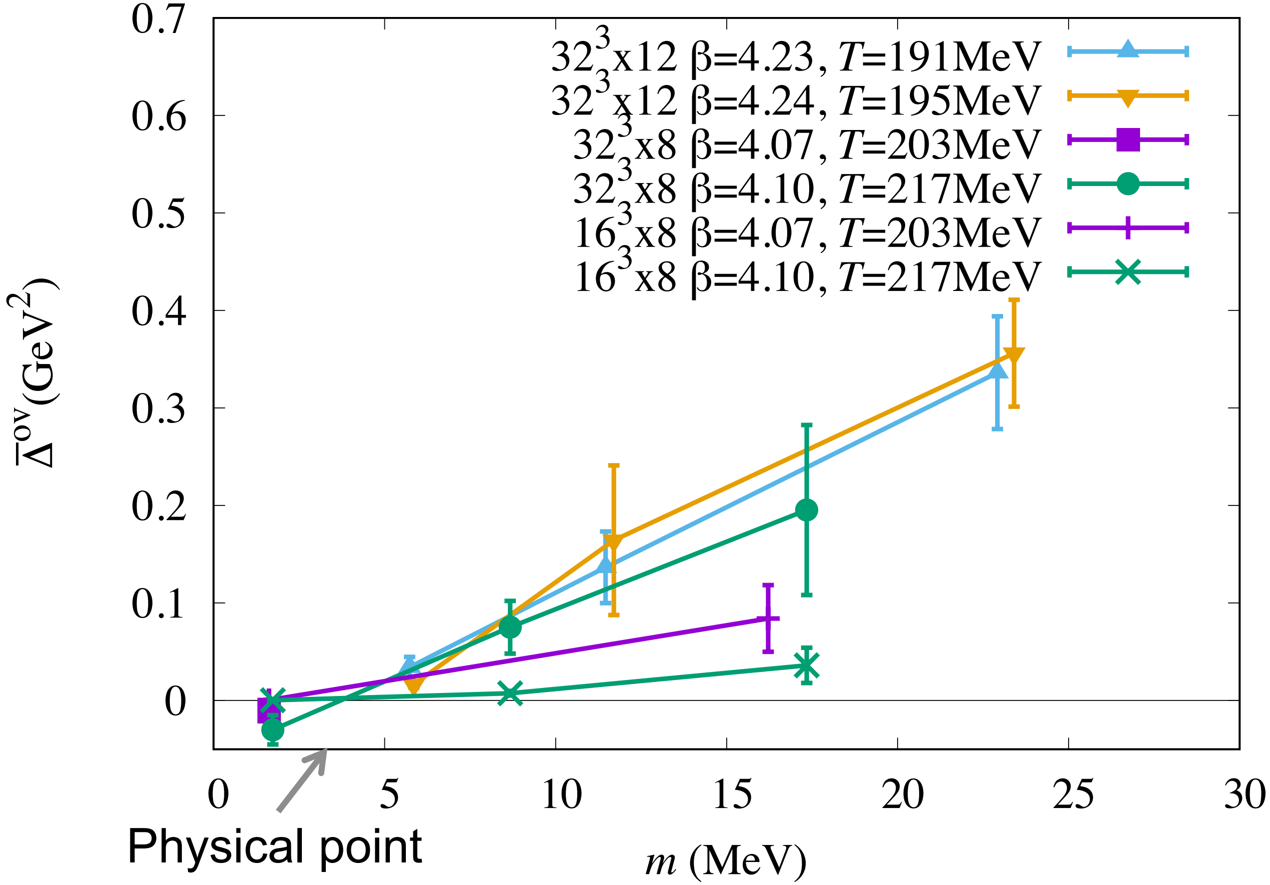
$$\bar{\Delta}^{\rm ov}_{\pi-\delta} \equiv \Delta^{\rm ov}_{\pi-\delta} - \frac{2N_0}{Vm^2} .$$

N_0 : # of zero modes $\quad (\sim 1/\sqrt{V})$



$U(1)_A$ ANOMALY VANISHES IN THE CHIRAL LIMIT

Coarse (a>0.08fm) lattice [JLQCD(Tomiya et al.) 2016]



T = 1.1 – 1.2 Tc

L=16 (1.8fm) and 32 (3.6fm) results are consistent.

$$(M_{screen} L > 5.)$$

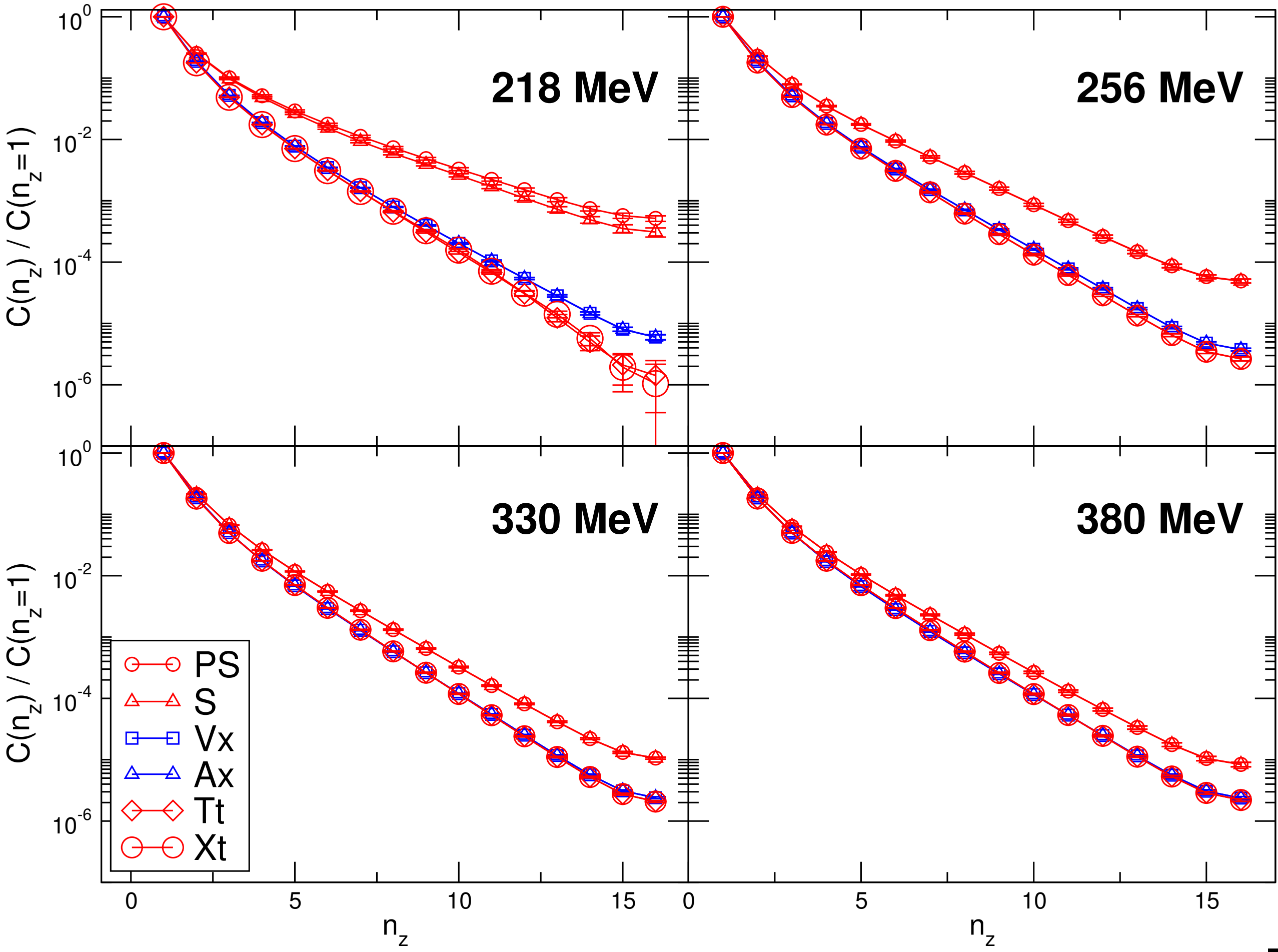
MESON CORRELATOR ITSELF SHOWS U(1) ANOMALY VANISHING

[C. Rohrhofer et al. 2017]

SU(2)xSU(2) [blue] and $U(1)_A$ (red) partners are degenerate.

[similar results reported by Brandt et al. 2016]

Further enhancement to SU(4) ? [Glozman 2015]



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$U(1)_A$ anomaly at T~1.1-1.4Tc (Tc~180MeV) in the chiral limit is consistent with zero.

4. **Result 2: topological susceptibility**

5. **Summary**

TOPOLOGICAL SUSCEPTIBILITY

$$\chi_t = \frac{\langle Q^2 \rangle}{V}$$

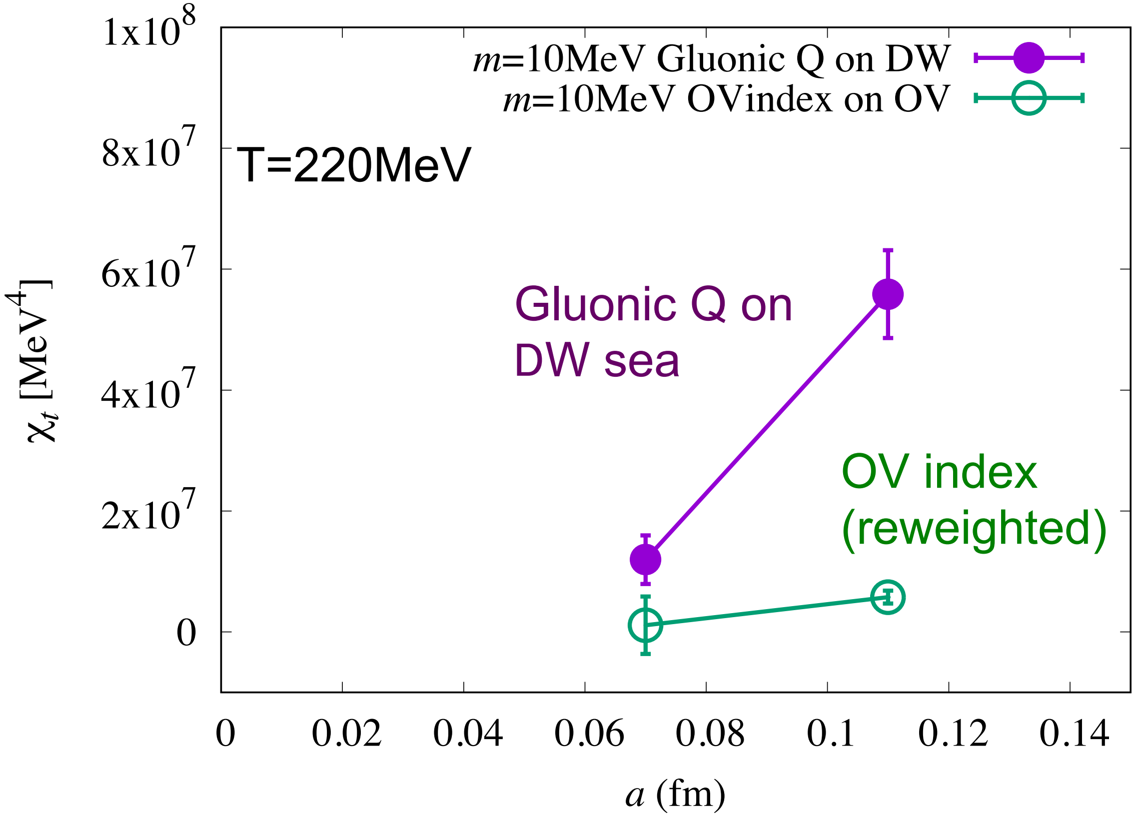
$$Q = \frac{1}{32\pi^2} \int d^4x \mathrm{Tr} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

another direct probe for U(1)$_A$ anomaly.

TOPOLOGICAL SUSCEPTIBILITY

Above Tc, it is sensitive to lattice artifact.

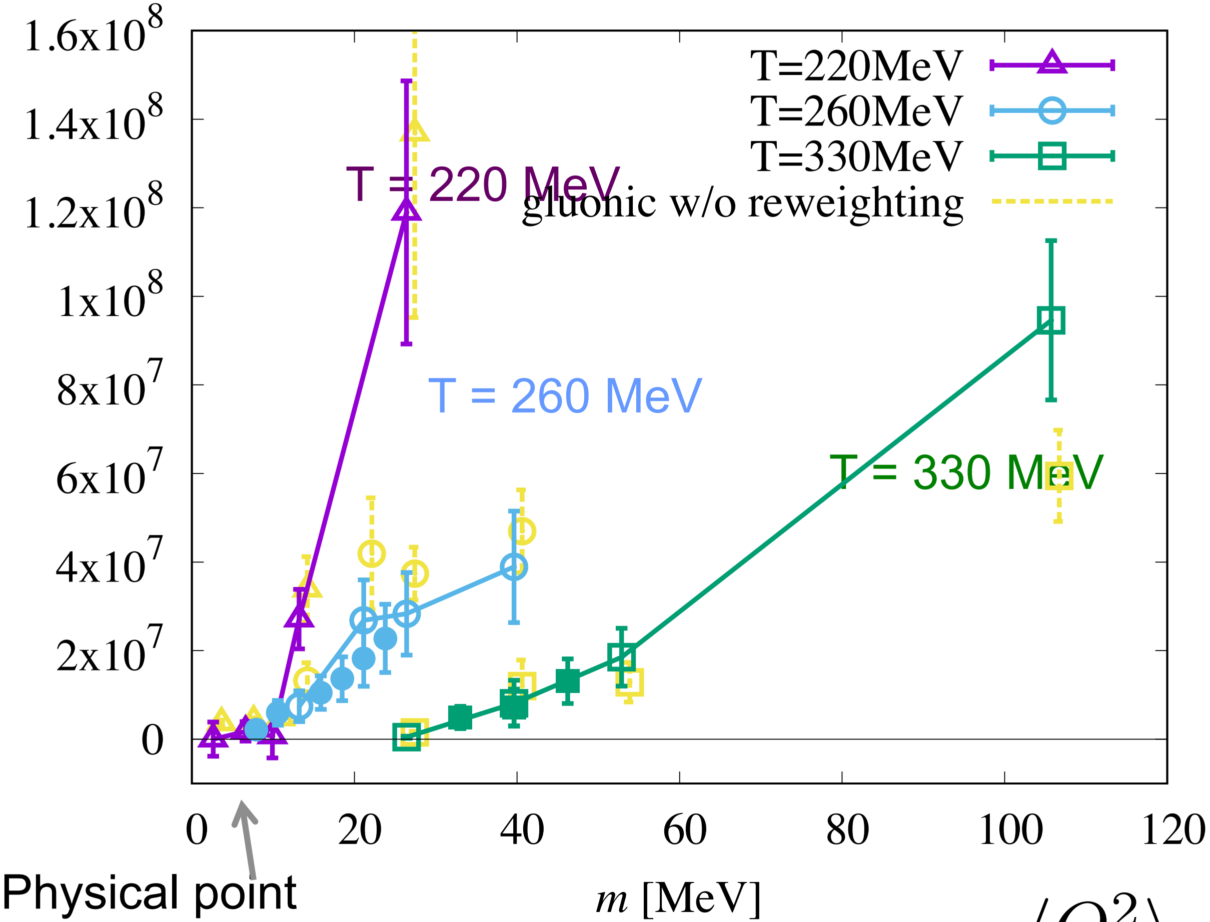
We need (reweighted) overlap fermion for a>0.08fm.



index of overlap Dirac operator is stable against lattice cut-off.

TOPOLOGICAL SUSCEPTIBILITY VANISHES *BEFORE* THE CHIRAL LIMIT

[JLQCD preliminary]



Physical point

L=48 (3.6fm) & L=32 (2.4fm) results are consistent.

On our fine lattices (a~0.07fm) OV index and gluonic def. after Wilson flow ($\sqrt{8t} \sim 0.47$ fm) are also consistent.

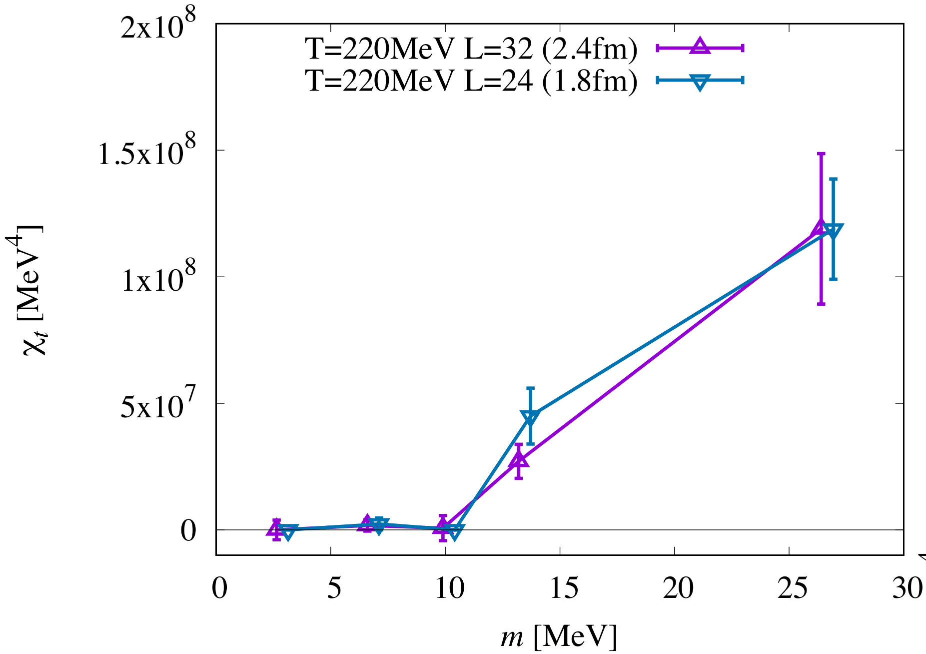
agrees with our prediction

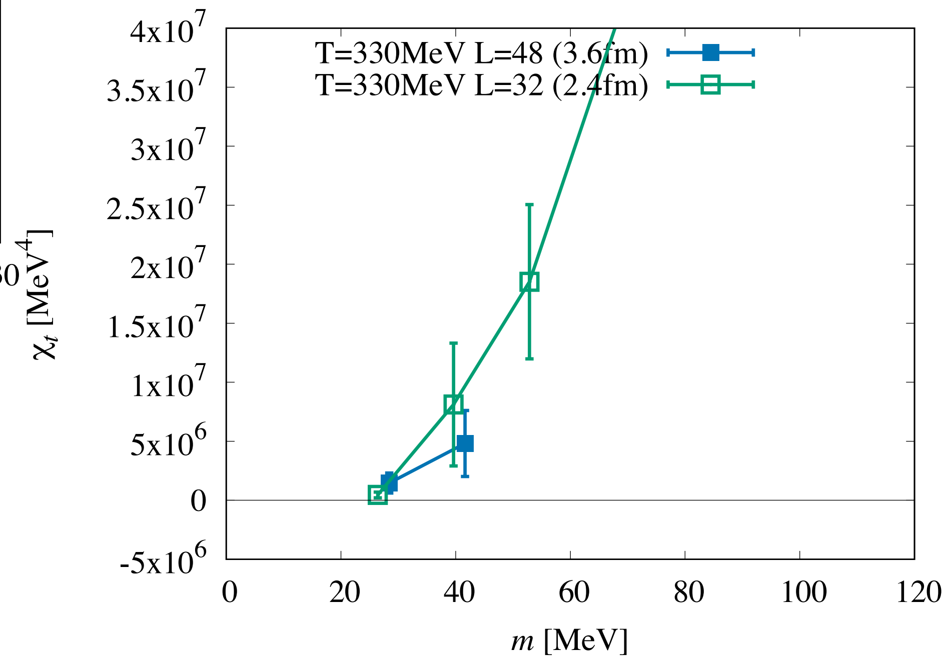
[Aoki, F, Taniguchi 2012]

$$\frac{\langle Q^2 \rangle}{V} = 0 \quad \text{for } m <^{\exists} m_{cr}$$

FINITE VOLUME DEPENDENCE

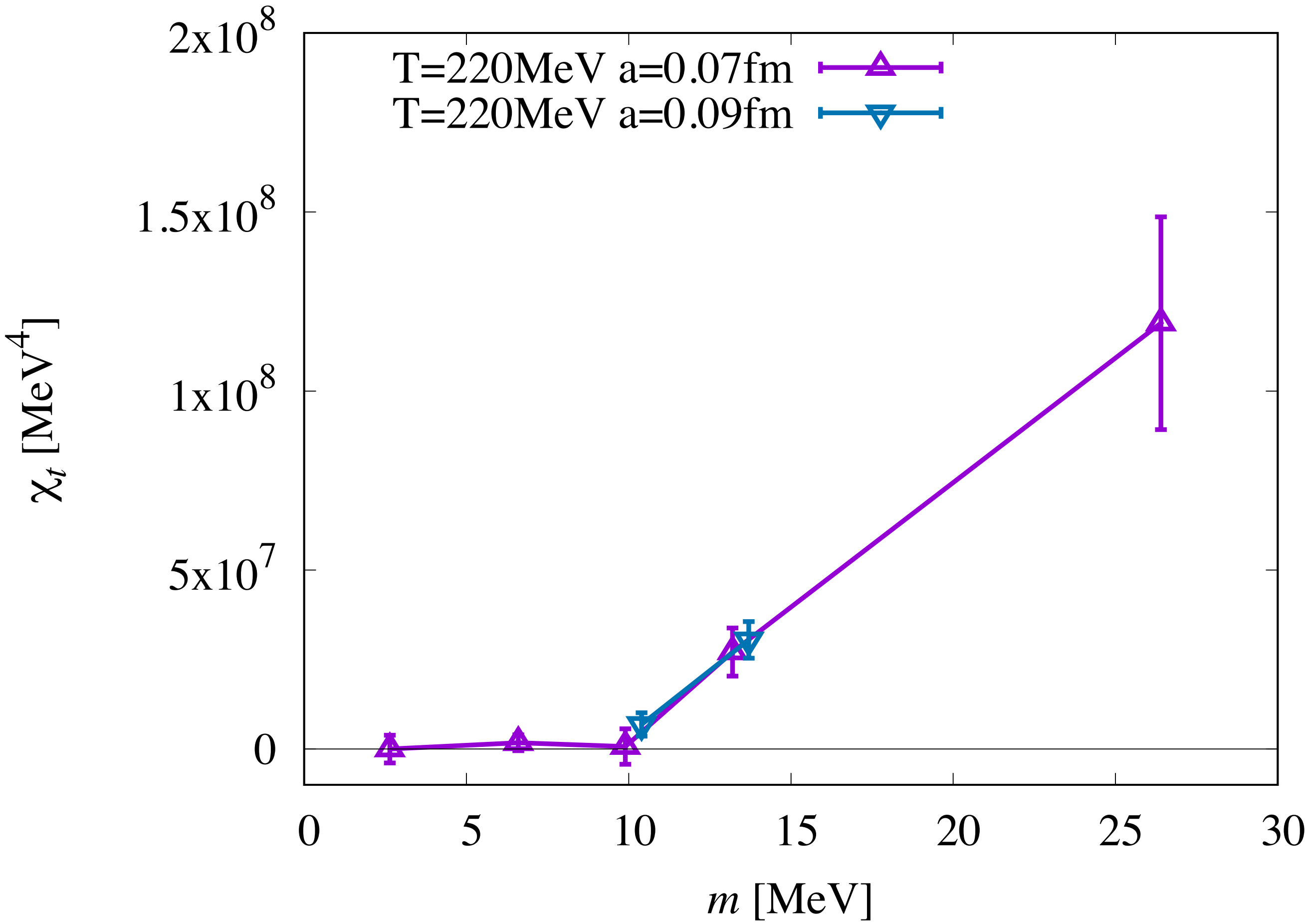
[JLQCD preliminary]





FINITE LATTICE SPACING DEPENDENCE

[JLQCD preliminary]



STRONG SUPPRESSION OF TOPOLOGICAL SUSCEPTIBILITY

If our data indicates

$$\frac{\langle Q^2 \rangle}{V} = 0 \quad \text{for } m <^{\exists} m_{cr}$$

Chiral phase transition is likely to be 1st order.

(There's no symmetry enhancement at finite quark mass.)

If $m_u, m_d < m_{cr},$ **there may be gravitational waves from QCD bubble collision in the early universe.**

CAN AXION BE A DARK MATTER?

If our result really indicates $\chi_t = 0$

and 1st order phase transition,

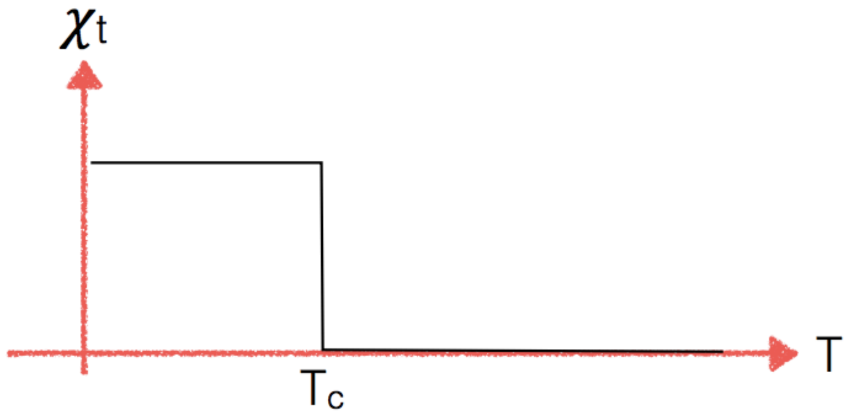


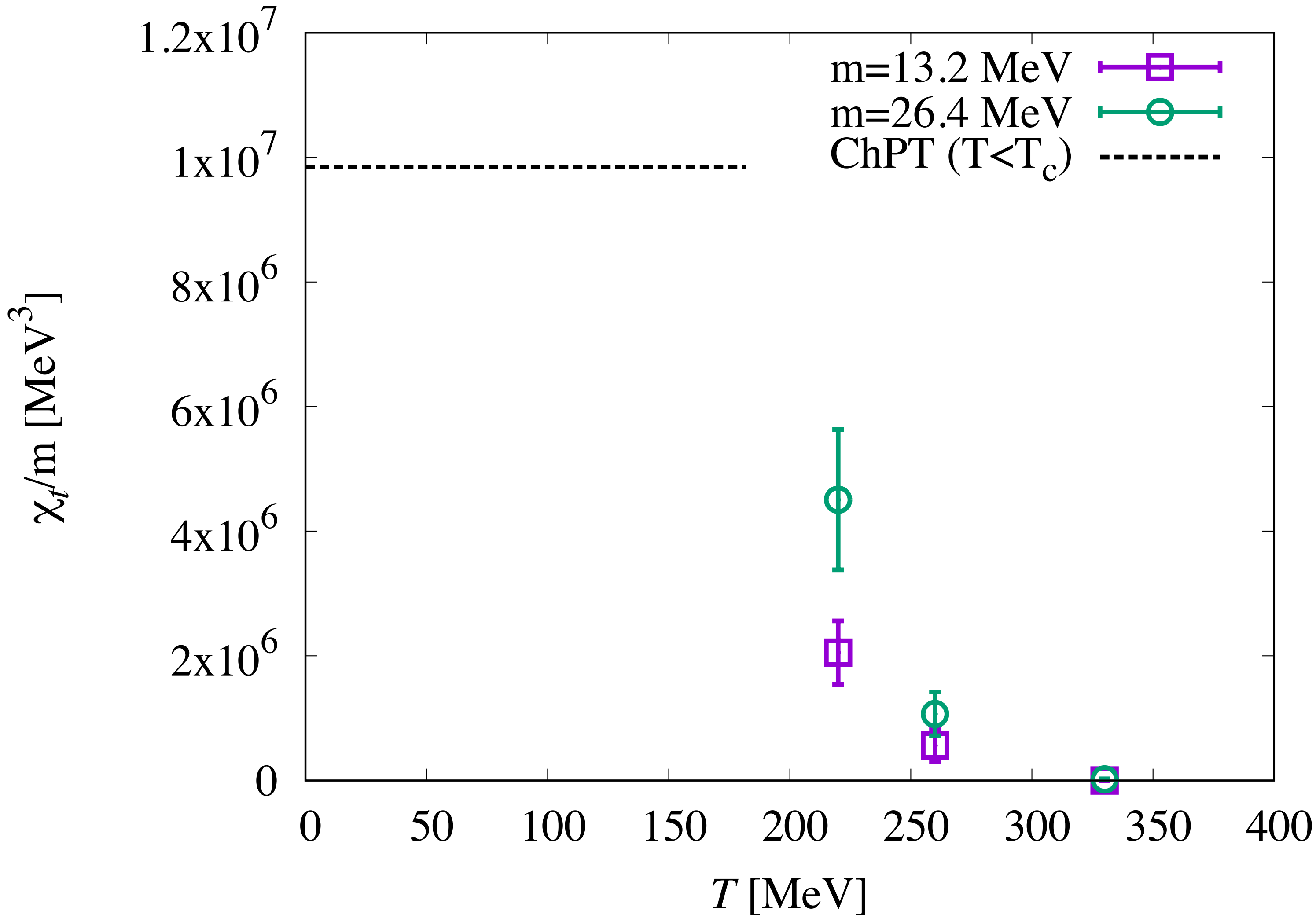
Figure from Kitano-san's seminar slide (2015)

$\Omega_a \sim 2 \times 10^5 \theta_{\rm ini}^2$ independent of m_a

Axion cannot be a dark matter since too much DM created (to expand our universe).

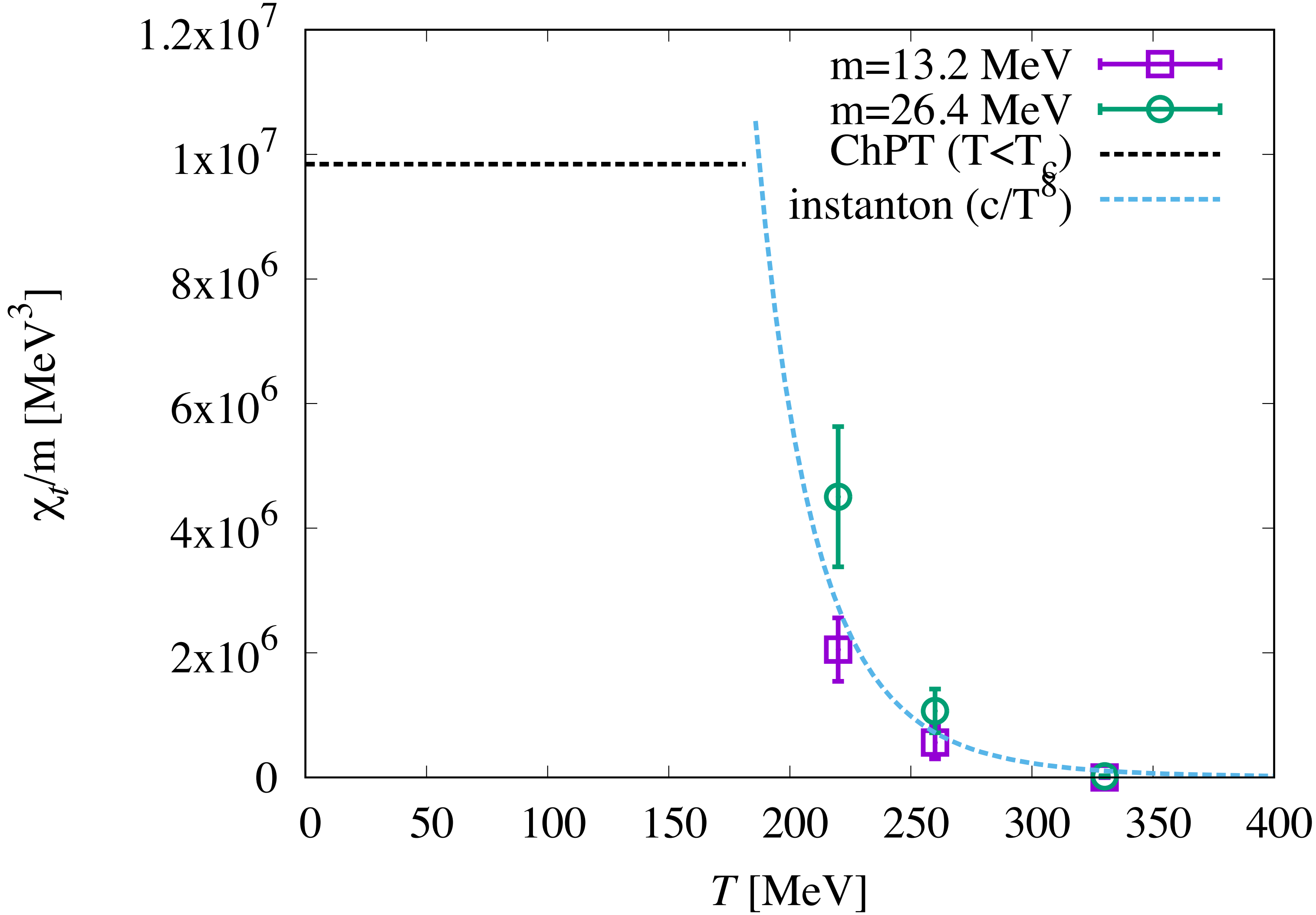
TEMPERATURE DEPENDENCE

Shows a sharp drop!

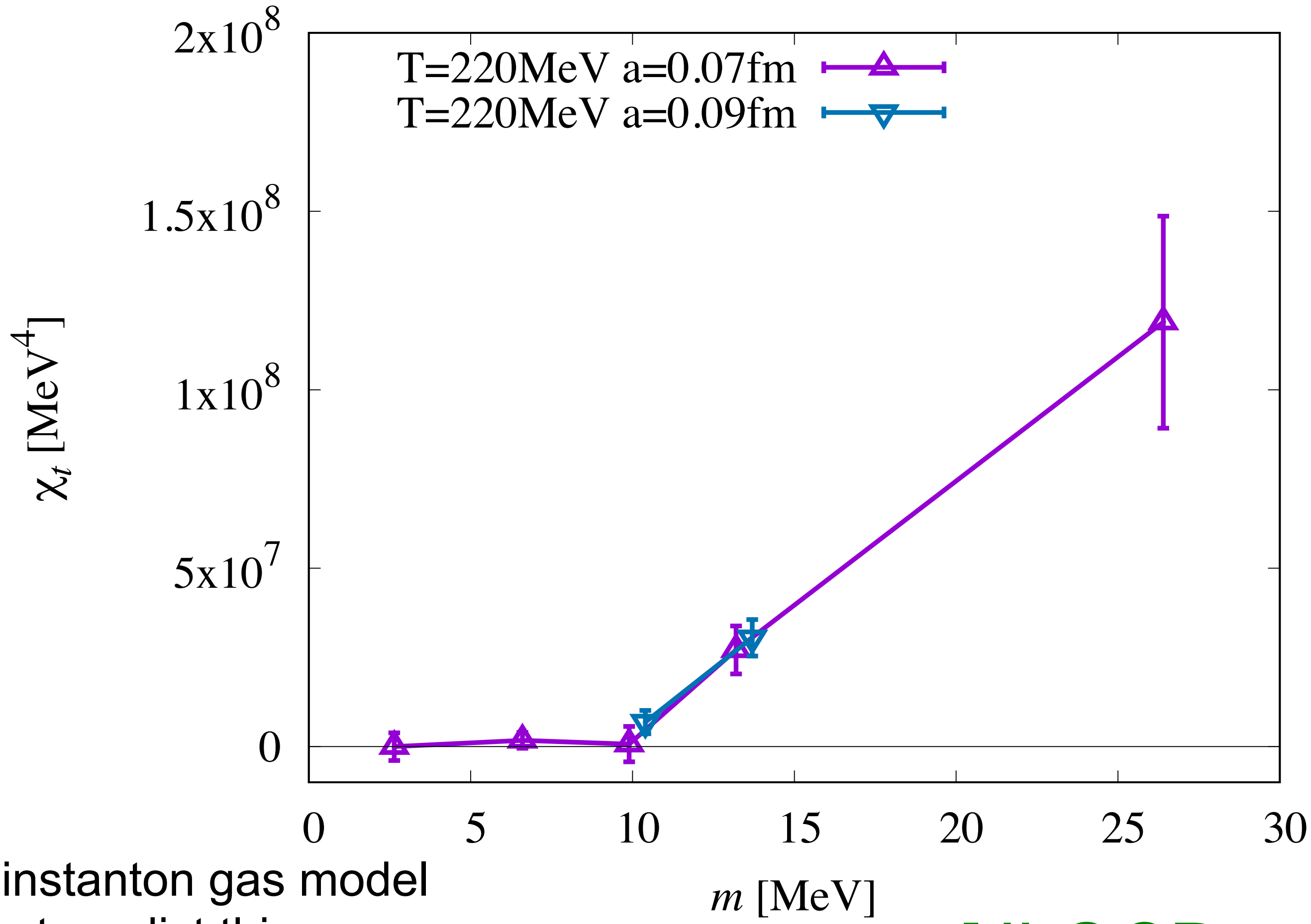


TEMPERATURE DEPENDENCE

But so does instanton model ($1/T^8$).



THE DROP IS STILL IMPRESSIVE.



Dilute instanton gas model does not predict this "discontinuity".

[JLQCD preliminary]

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Our answer = YES. $SU(2)_L x SU(2)_R$ and $U(1)_A$ are connected through Dirac spectrum.

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We need good chiral sym (or careful cont. limit.).

✔ 3. **Result 1: $U(1)_A$ anomaly**

$U(1)_A$ anomaly at T~1.1-1.4Tc (Tc~180MeV)
in the chiral limit is consistent with zero.

✔ 4. **Result 2: topological susceptibility**

Topological susceptibility drops *before* the chiral limit.

5. **Summary**

SUMMARY

1. $U(1)_A$ anomaly at high T is a non-trivial problem.
2. $U(1)_A$ and $SU(2)_L x SU(2)_R$ order prms. connected.
3. $U(1)_A$ is sensitive to lattice artifact at high T
   -> We need overlap fermion for a>0.08 fm.

   For a=0.07 fm, Mobius domain-wall is O.K.
4. In our simulation with chiral fermions at 3 volumes and 3-10 quark masses at T=1.1-1.8Tc (Tc~180MeV), $U(1)_A$ anomaly disappears
   [before the chiral limit]
   (suggesting 1st order transition ?).

MAIN MESSAGE OF THIS TALK

In high T QCD, whether

$$\left\langle \langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} - \langle \delta_A O(x) \rangle_{fermion} \delta(x - x') \right\rangle_{gluons}$$

$$= \left\langle \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion} \right\rangle_{gluons} = 0???$$

or not is a non-trivial question, which can only be answered by carefully integrating over gluons (by lattice QCD).

BACK UP SLIDES

SUM OF NON-ZERO QUANTITY NONZERO ?

NOT always.

Example: chiral condensate

$$\langle \bar{q}q \rangle = \frac{\int dA \mathrm{Tr} D^{-1} \det D e^{-S_G}}{Z}$$

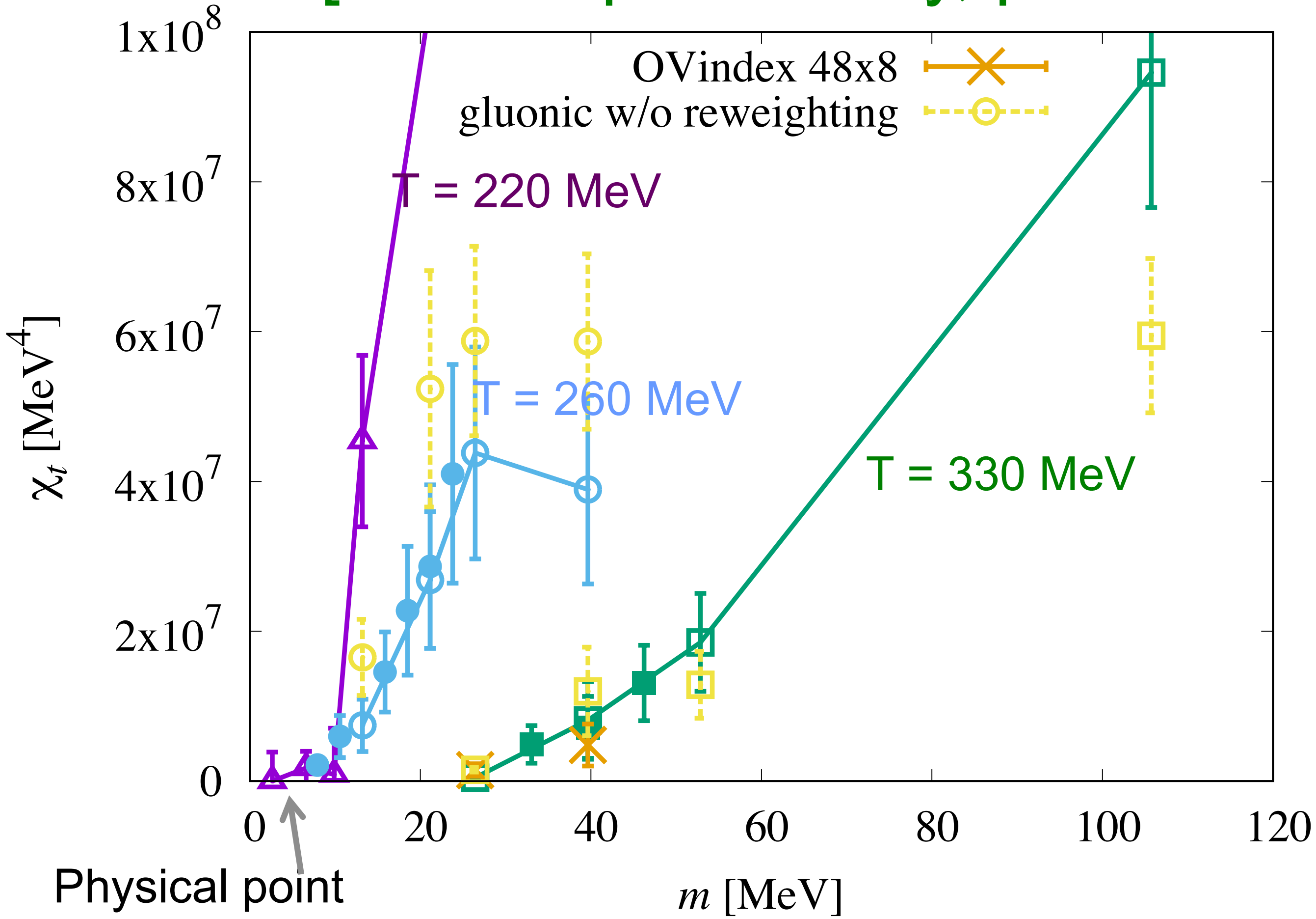
can be zero and non-zero. How about

$$\langle \langle \partial_\mu J_5^\mu (x) O(x') \rangle_{fermion} - \langle \delta_A O(x) \rangle_{fermion} \delta(x - x') \rangle_{gluons}$$

$$= \left\langle \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion} \right\rangle_{gluons} = 0???$$

SHARP DROP IS NOT DUE TO TOPOLOGY FREEZING

[JLQCD preliminary, parallel talk by Y. Aoki]



Filled symbols = mass reweighting from heavier mass, where topology is fluctuating well.

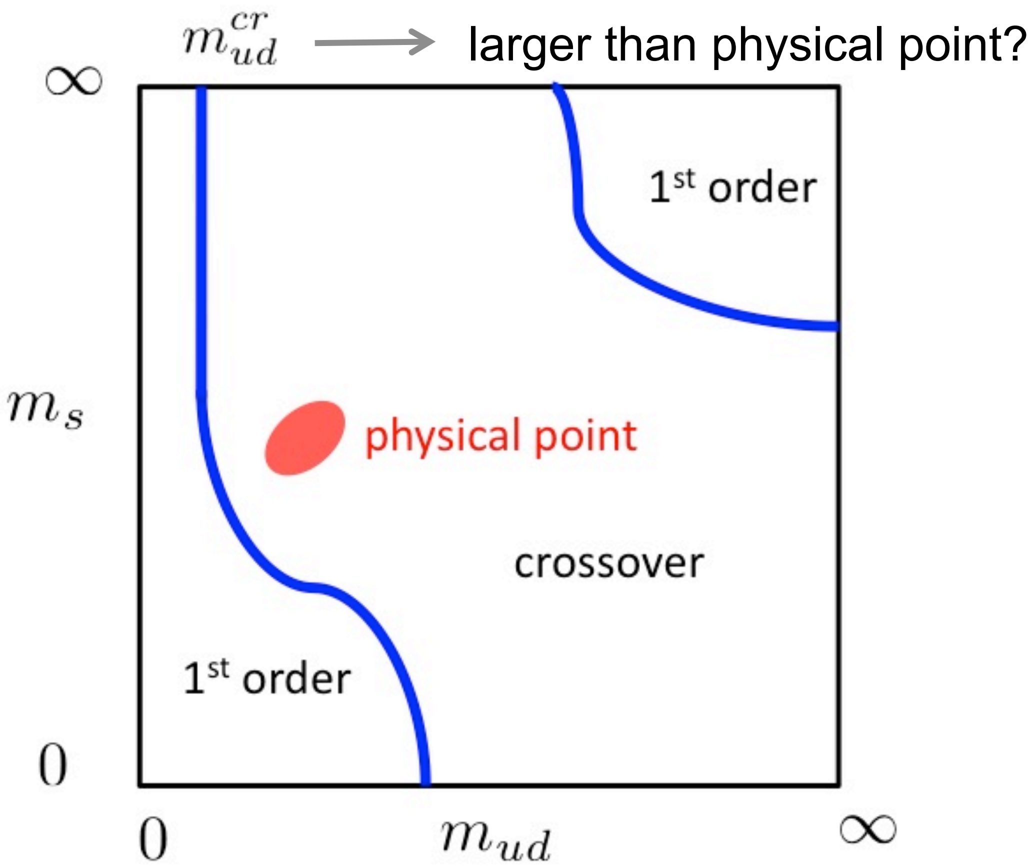
WHY BAD MODES ONLY ABOVE T_c?

Suppose bad modes are always there but sparse.

At T=0, they mix with MANY good low-lying modes, then relative lattice artifact is comparable to residual mass.

At T>Tc, good modes are also SPARSE, the lattice artifacts remain large.

PHASE DIAGRAM?

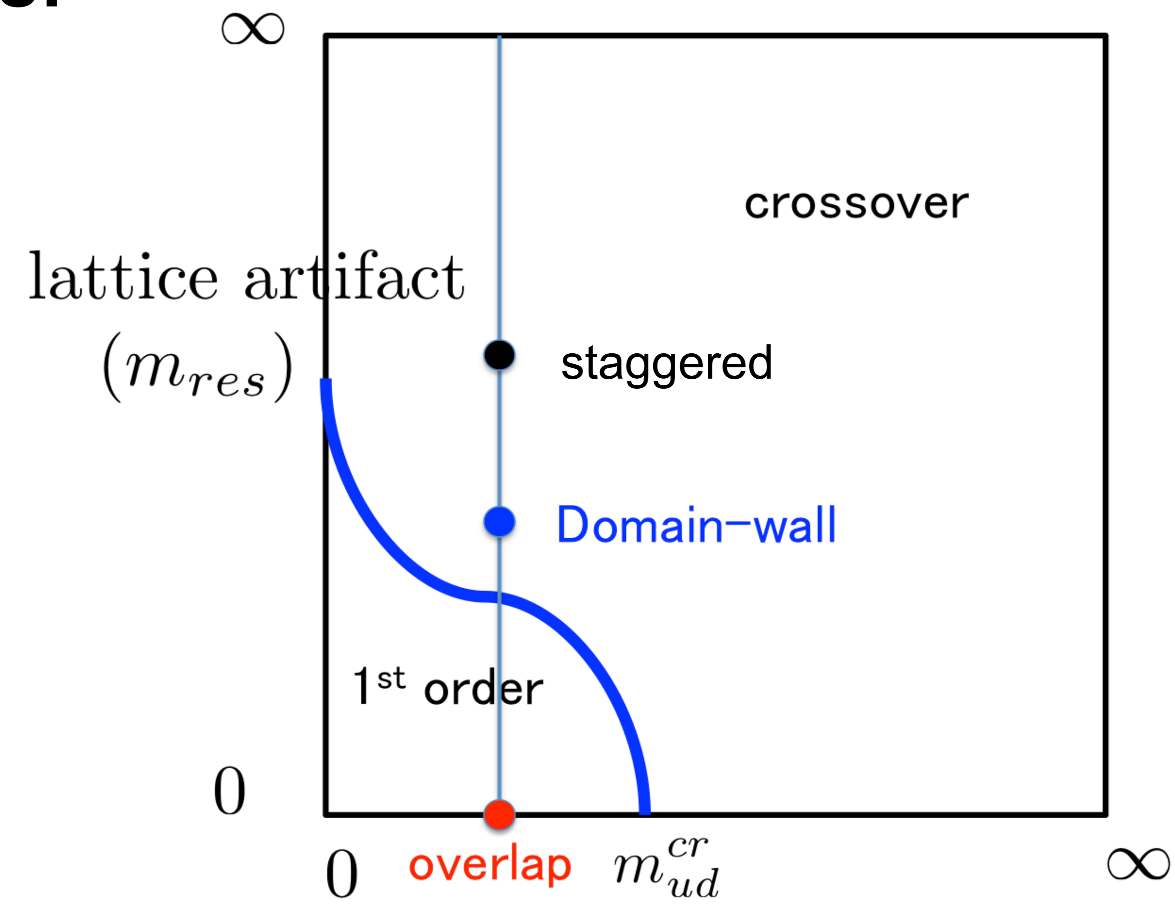


WHY DIFFERENT ?

Overlap and non-chiral fermions may be in different phases:

Lattice artifact

= another axis

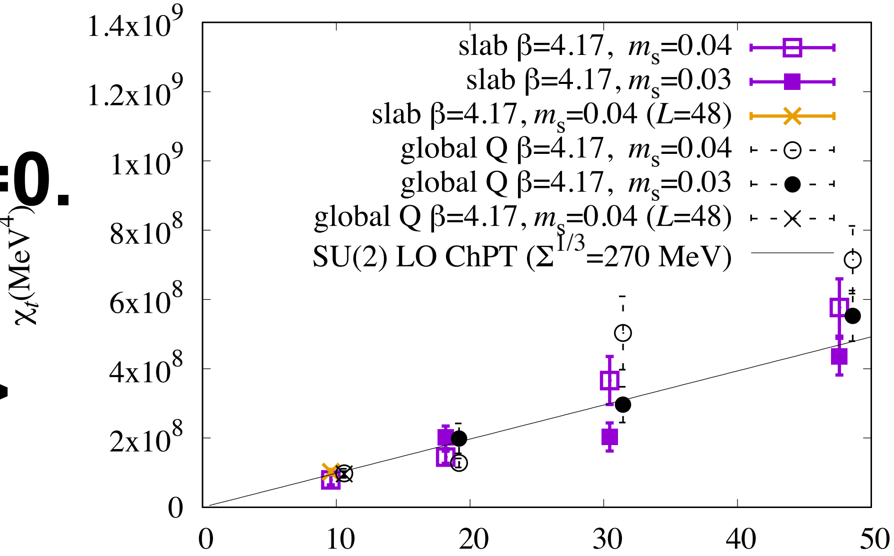


WHEN WE USE DOMAIN-WALL FERMIONS WE MUST

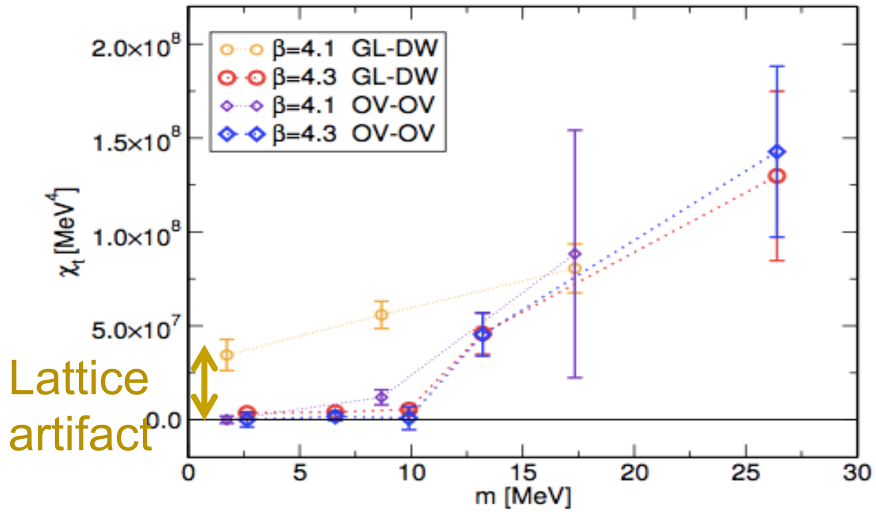
1. Check mass dependence at T=0.

2. Check mass dependence at T> Tc : if m=0 limit is consistent with zero.

Otherwise, your results could be contaminated by lattice artifacts.



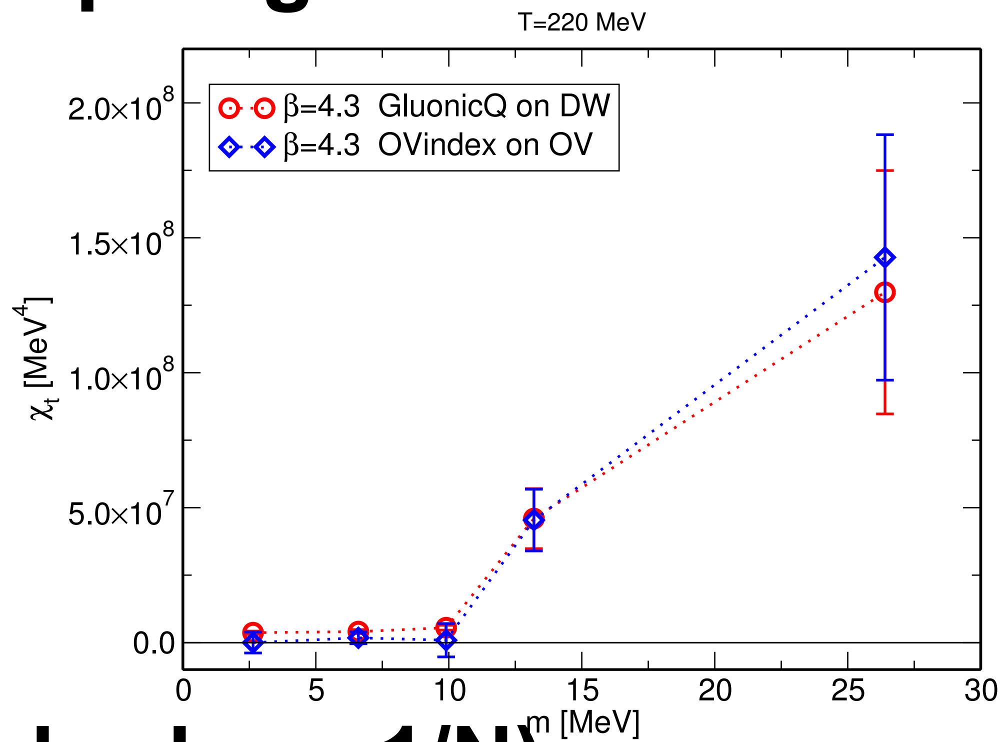
compare N_t=8(β=4.1) and 12(4.3) at simlar temperature (217 and 220 MeV)



RESOLUTION OF <Q²>

Topological charge is discrete.

→ resolution of topological susceptibility is limitted by # of confs.



We put max(standard err, 1/N) to estimate the statistical error.

ASSUMPTIONS IN AOKI-FUKAYA-TANIGUCHI 2012

1. SU(2) x SU(2) fully recovered at Tc.
2. if $\mathcal{O}(A)$ is m-independent

   $\langle \mathcal{O}(A) \rangle_m = f(m^2)$ $\quad$ $f(x)$ is analytic at $x=0$

3. if $\mathcal{O}(A)$ is m-independent and positive, and satisfies

$$\lim_{m\to 0} \frac{1}{m^{2k}} \langle \mathcal{O}(A) \rangle_m = 0$$

$$\Longrightarrow \langle \mathcal{O}(A) \rangle_m = m^{2(k+1)} \underbrace{\int \mathcal{D}A\, \hat{P}(m^2, A) \mathcal{O}(A)}_{\text{finite}}$$

$$\Longrightarrow \langle \mathcal{O}(A)^l \rangle_m = m^{2(k+1)} \int \mathcal{D}A\, \hat{P}(m^2, A) \mathcal{O}(A)^l = O(m^{2(k+1)})$$

4. $\rho^A(\lambda) \equiv \lim_{V\to\infty} \frac{1}{V} \sum_n \delta\left(\lambda - \sqrt{\bar{\lambda}_n^A \lambda_n^A}\right) = \sum_{n=0}^{\infty} \rho_n^A \frac{\lambda^n}{n!}$ at $\lambda = 0$ $(\lambda < \epsilon)$

(4 can be removed.)

OUR OVERLAP DIRAC OPERATOR

$$D_{\text{ov}}(m) = \sum_{|\lambda_i^M|<\lambda_{\text{th}}^M} \left[\frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \,\text{sgn}(\lambda_i^M)\right] |\lambda_i^M\rangle\langle\lambda_i^M|$$

$$+ D_{\text{DW}}^{\text{4D}}(m) \left[1 - \sum_{\lambda_i^M<|\lambda_{\text{th}}^M|} |\lambda_i^M\rangle\langle\lambda_i^M|\right],$$

λ_i^M : eigenvalue of H_M.

"EFFICIENCY" OF OV/DW REWEIGHTING

$$\frac{N_{eff}}{N} = \frac{\langle R \rangle}{N max(R)}$$

On our 2-4 fm lattices at T=1.1-1.8Tc (Tc~180MeV)

a ~ 0.1 fm : O.K. for L=2 fm, $N_{eff}/N \sim 1/20$

but does not work for 4 fm. $N_{eff}/N < 1/1000.$

(→ we approximate it by O(10) low-modes.)

a ~ 0.08 fm : works well (3 fm). $N_{eff}/N \sim 1/10$

a ~ 0.07 fm : domain-wall & overlap are consistent (2.4, 3.6 fm). $N_{eff}/N > 1/10$

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Our focus in this talk